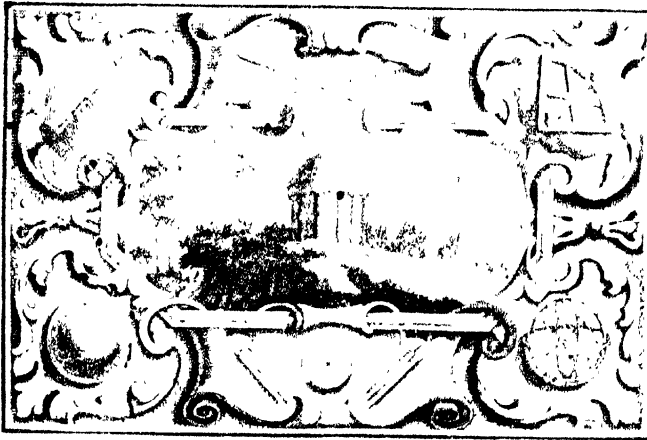


V I E W

O F

Sir *ISAAC NEWTON*'s

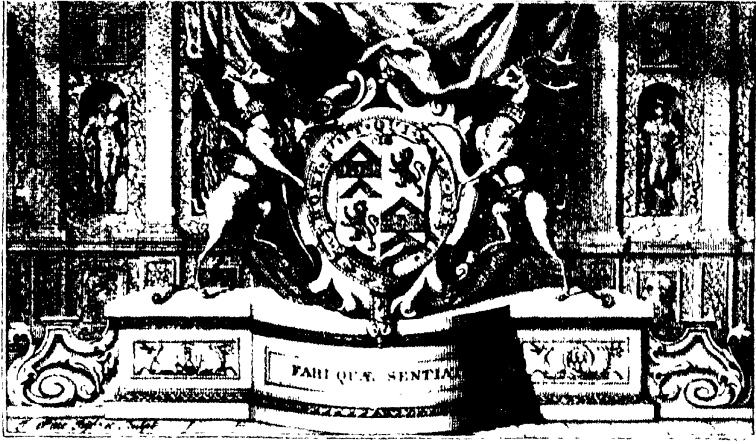
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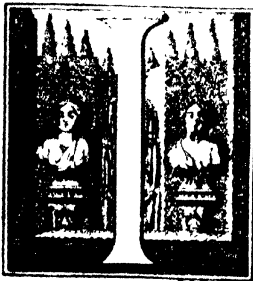
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To the Noble and Right Honourable
SIR ROBERT WALPOLE.

SIR,



Take the liberty to send you this view of Sir ISAAC NEWTON's philosophy, which, if it were performed suitable to the dignity of the subject, might not be a present unworthy the acceptance of the greatest person. For his philosophy affords us the only true account of the

DEDICATION.

operations of nature, which for so many ages had imployed the curiosity of mankind ; though no one before him was furnished with the strength of mind necessary to go any depth in this difficult search. However, I am encouraged to hope, that this attempt, imperfect as it is, to give our countrymen in general some conception of the labours of a person, who shall always be the boast of this nation, may be received with indulgence by one, under whose influence these kingdoms enjoy so much happiness. Indeed my admiration at the surprizing inventions of this great man, carries me to conceive of him as a person, who not only must raise the glory of the country, which gave him birth ; but that he has even done honour to human nature, by having extended the greatest and most noble of our faculties, reason, to subjects, which, till he attempted them, appeared to be wholly beyond the reach of our limited capacities. And what can give us any more
more

DEDICATION.

more pleasing prospect of our own condition, than to see so exalted a proof of the strength of that faculty, whereon the conduct of our lives, and our happiness depends; our passions and all our motives to action being in such manner guided by our opinions, that where these are just, our whole behaviour will be praise-worthy? But why do I presume to detain you, SIR, with such reflections as these, who must have the fullest experience within your own mind, of the effects of right reason? For to what other source can be ascribed that amiable frankness and unreserved condescension among your friends, or that masculine perspicuity and strength of argument, whereby you draw the admiration of the publick, while you are engaged in the most important of all causes, the liberties of mankind?

I humbly crave leave to make the only acknowledgement within my power, for the benefits,
which

DEDICATION.

which I receive in common with the rest of my countrymen from these high talents, by subscribing my self

S I R,

Your most faithful,

and

Most humble Servant,

HENRY PEMBERTON.

P R E F A C E.

I Drew up the following papers many years ago at the desire of some friends, who, upon my taking care of the late edition of Sir ISAAC NEWTON's *Principia*, perswaded me to make them publick. I laid hold of that opportunity, when my thoughts were afresh employed on this subject, to revise what I had formerly written. And I now send it abroad not without some hopes of answering these two ends. My first intention was to convey to such, as are not used to mathematical reasoning, some idea of the philosophy of a person, who has acquired an universal reputation, and rendered our nation famous for these speculations in the learned world. To which purpose I have avoided using terms of art as much as possible, and taken care to define such as I was obliged to use. Though this caution was the less necessary at present, since many of them are become familiar words to our language, from the great number of books wrote in it upon philosophical subjects, and the courses of experiments, that have of late years been given by several ingenious men. The other view I had, was to encourage such young gentlemen as have a turn for the mathematical sciences, to pursue those studies the more chearsfully, in order to understand in our author himself the demonstrations of the things I here declare. And to facilitate their progress herein, I intend to proceed still farther in the explanation of Sir ISAAC NEWTON's philosophy. For as I have received very much pleasure from perusing his writings, I hope it is no illaudable ambition to endeavour the rendering them more easily understood, that greater numbers may enjoy the same satisfaction.

It will perhaps be expected, that I should say something particular of a person, to whom I must always acknowledge my self to be much obliged. What I have to declare on this head will be but short; for it was in the very last years of Sir ISAAC's life, that I had the ho-

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nour of his acquaintance. This happened on the following occasion. Mr. Polenus, a Professor in the University of Padua, from a new experiment of his, thought the common opinion about the force of moving bodies was overturned, and the truth of Mr. Libnitz's notion in that matter fully proved. The contrary of what Polenus had asserted I demonstrated in a paper, which Dr. MEAD, who takes all opportunities of obliging his friends, was pleased to shew Sir ISAAC NEWTON. This was so well approved of by him, that he did me the honour to become a fellow-writer with me, by annexing to what I had written, a demonstration of his own drawn from another consideration. When I printed my discourse in the philosophical transactions, I put what Sir ISAAC had written in a scholium by it self, that I might not seem to usurp what did not belong to me. But I concealed his name, not being then sufficiently acquainted with him to ask whether he was willing I might make use of it or not. In a little time after he engaged me to take care of the new edition he was about making of his Principia. This obliged me to be very frequently with him, and as he lived at some distance from me, a great number of letters passed between us on this account. When I had the honour of his conversation, I endeavoured to learn his thoughts upon mathematical subjects, and something historical concerning his intentions, that I had not been before acquainted with. I found, he had read fewer of the modern mathematicians, than one could have expected; but his own prodigious invention readily supplied him with what he might have an occasion for in the pursuit of any subject he undertook. I have often heard him censure the handling geometrical subjects by algebraic calculations, and his book of Algebra he called by the name of Universal Arithmetic, in opposition to the injudicious title of Geometry, which Des Cartes had given to the treatise, wherein he shews, how the geometer may assist his invention by such kind of computations. He frequently praised Slusius, Barrow and Huygens for not being influenced by the false taste, which then began to prevail. He used to commend the laudable attempt of Hugo de Omerique to restore the ancient analysis, and very much esteemed Apollonius's book De sectione rationis for giving us a clearer notion of that analysis than we had before. Dr. Barrow may be esteemed as hav-

ing

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ing shewn a compass of invention equal, if not superior to any of the moderns, our author only excepted; but Sir ISAAC NEWTON has several times particularly recommended to me Huygens's stile and manner. He thought him the most elegant of any mathematical writer of modern times, and the most just imitator of the ancients. Of their taste, and form of demonstration Sir ISAAC always professed himself a great admirer. I have heard him even censure himself for not following them yet more closely than he did; and speak with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of Des Cartes and other algebraic writers, before he had considered the elements of Euclid with that attention, which so excellent a writer deserves. As to the history of his inventions, what relates to his discoveries of the methods of series and fluxions, and of his theory of light and colours, the world has been sufficiently informed of already. The first thoughts, which gave rise to his Principia, he had, when he retired from Cambridge in 1666 on account of the plague. As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not sensibly diminished at the remotest distance from the center of the earth, to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude, that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself? and if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance, at which we can place our selves from the center of the earth; yet it is very possible, that so high as the moon this power may differ much in strength from what it is here. To make an estimate, what might be the degree of this diminution, he considered with himself, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power. And by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must decrease in the duplicate proportion of the increase of distance. This

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be concluded by supposing them to move in perfect circles concentrical to the sun, from which the orbits of the greatest part of ~~them~~ do not much differ. Supposing therefore the power of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit. In this computation, being absent from books, he took the common estimate in use among geographers and our seamen, before Norwood had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the earth. But as this is a very faulty supposition, each degree containing about $69\frac{1}{2}$ of our miles, his computation did not answer expectation; whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. On this account he laid aside for that time any farther thoughts upon this matter. But some years after, a letter which he received from Dr. Hook, put him on inquiring what was the real figure, in which a body let fall from any high place descends, taking the motion of the earth round its axis into consideration. Such a body, having the same motion, which by the revolution of the earth the place has whence it falls, is to be considered as projected forward and at the same time drawn down to the center of the earth. This gave occasion to his resuming his former thoughts concerning the moon; and Picart in France having lately measured the earth, by using his measures the moon appeared to be kept in her orbit purely by the power of gravity; and consequently, that this power decreases as you recede from the center of the earth in the manner our author had formerly conjectured. Upon this principle he found the line described by a falling body to be an ellipsis, the center of the earth being one focus. And the primary planets moving in such orbits round the sun, he had the satisfaction to see, that this inquiry, which he had undertaken merely out of curiosity, could be applied to the greatest purposes. Hereupon he composed near a dozen propositions relating to the motion of the primary planets about the sun. Several years after this, some discourse he had with Dr. Halley, who at Cambridge made him a visit, engaged Sir ISAAC NEWTON to resume again the consideration of this subject; and gave occasion

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to his writing the treatise which he published under the title of mathematical principles of natural philosophy. This treatise, full of such a variety of profound inventions, was composed by him from scarce any other materials than the few propositions before mentioned, in the space of one year and an half.

Though his memory was much decayed, I found he perfectly understood his own writings, contrary to what I had frequently heard in discourse from many persons. This opinion of theirs might arise perhaps from his not being always ready at speaking on these subjects, when it might be expected he should. But as to this, it may be observed, that great genius's are frequently liable to be absent, not only in relation to common life, but with regard to some of the parts of science they are the best informed of. Inventors seem to treasure up in their minds, what they have found out, after another manner than those do the same things, who have not this inventive faculty. The former, when they have occasion to produce their knowledge, are in some measure obliged immediately to investigate part of what they want. For this they are not equally fit at all times: so it has often happened, that such as retain things chiefly by means of a very strong memory, have appeared off hand more expert than the discoverers themselves.

As to the moral endowments of his mind, they were as much to be admired as his other talents. But this is a field I leave others to expatiate in. I only touch upon what I experienced myself during the few years I was happy in his friendship. But this I immediately discovered in him, which at once both surprized and charmed me: Neither his extreme great age, nor his universal reputation had rendered him stiff in opinion, or in any degree elated. Of this I had occasion to have almost daily experience. The Remarks I continually sent him by letters on his Principia were received with the utmost goodness. These were so far from being any ways displeasing to him, that on the contrary it occasioned him to speak many kind things of me to my friends, and to honour me with a publick testimony of his good opinion. He also approved of the following treatise, a great part of which we read together. As many alterations were

A POEM ON SIR ISAAC NEWTON.

From black obscurity's abyss to raise,
(Drooping and mourning o'er thy wondrous works)
With vain inquiry fought. Like meteors these
In their dark age bright sons of wisdom shone :
But at thy NEWTON all their laurels fade,
They shrink from all the honours of their names.
So glimm'ring stars contract their feeble rays,
When the swift lustre of AURORA'S face
Flows o'er the skies, and wraps the heav'ns in light.

THE Deity's omnipotence, the cause,
Th' original of things long lay unknown.
Alone the beauties prominent to fight
(Of the celestial power the outward form)
Drew praise and wonder from the gazing world.
As when the deluge overspread the earth,
Whilst yet the mountains only rear'd their heads
Above the surface of the wild expanse,
Whelm'd deep below the great foundations lay,
Till some kind angel at heav'n's high command
Roul'd back the rising tides, and haughty floods,
And to the ocean thunder'd out his voice :
Quick all the swelling and imperious waves,
The foaming billows and obscuring surge,
Back to their channels and their ancient seats
Recoil affrighted : from the darksome main
Earth raises smiling, as new-born, her head,
And with fresh charms her lovely face arrays.
So his extensive thought accomplish'd first
The mighty task to drive th' obstructing mists
Of ignorance away, beneath whose gloom
Th' inshrouded majesty of Nature lay.
He drew the veil and swell'd the spreading scene.
How had the moon around th' ethereal void

Rang'd,

A POEM ON SIR ISAAC NEWTON.

Rang'd, and eluded lab'ring mortals care,
Till his invention trac'd her secret steps,
While she inconstant with unsteady rein
Through endless mazes and meanders guides
In its unequal course her changing carr :
Whether behind the sun's superior light
She hides the beauties of her radiant face,
Or, when conspicuous, smiles upon mankind,
Unveiling all her night-rejoicing charms.
When thus the silver-tressed moon dispels
The frowning horrors from the brow of night,
And with her splendors cheers the fullen gloom,
While sable-mantled darkness with his veil
The visage of the fair horizon shades,
And over nature spreads his raven wings ;
Let me upon some unfrequented green
While sleep sits heavy on the drowsy world,
Seek out some solitary peaceful cell,
Where darksome woods around their gloomy brows
Bow low, and ev'ry hill's protended shade
Obscures the dusky vale, there silent dwell,
Where contemplation holds its still abode,
There trace the wide and pathless void of heav'n,
And count the stars that sparkle on its robe.
Or else in fancy's wild'ring mazes lost
Upon the verdure see the fairy elves
Dance o'er their magick circles, or behold,
In thought enraptur'd with the ancient bards,
Medea's baleful incantations draw
Down from her orb the paly queen of night.
But chiefly NEWTON let me soar with thee,
And while surveying all yon starry vault
With admiration I attentive gaze,
Thou shalt descend from thy celestial seat,

And

A POEM on Sir ISAAC NEWTON.

And wait aloft my high-aspiring mind,
Shalt shew me there how nature has ordain'd
Her fundamental laws, shalt lead my thought
Through all the wand'rings of th' uncertain moon,
And teach me all her operating powers.
She and the sun with influence conjoint
Wield the huge axle of the whirling earth,
And from their just direction turn the poles,
Slow urging on the progress of the years.
The constellations seem to leave their seats,
And o'er the skies with solemn pace to move.
You, splendid rulers of the day and night,
The seas obey, at your resistless sway
Now they contract their waters, and expose
The dreary desert of old ocean's reign.
The craggy rocks their horrid sides disclose ;
Trembling the sailor views the dreadful scene,
And cautiously the threat'ning ruin shuns.
But where the shallow waters hide the sands,
There ravenous destruction lurks conceal'd,
There the ill-guided vessel falls a prey, '—
And all her numbers gorge his greedy jaws.
But quick returning see th' impetuous tides
Back to th' abandon'd shores impell the main.
Again the foaming seas extend their waves,
Again the rouling floods embrace the shoars,
And veil the horrors of the empty deep.
Thus the obsequious seas your power confess,
While from the surface healthful vapours rise
Plenteous throughout the atmosphere diffus'd,
Or to supply the mountain's heads with springs,
Or fill the hanging clouds with needful rains,
That friendly streams, and kind refreshing show'rs
May gently lave the sun-burnt thirsty plains,

A POEM on Sir ISAAC NEWTON.

Or to replenish all the empty air
With wholesome moisture to increase the fruits
Of earth, and bless the labours of mankind.
O NEWTON, whether flies thy mighty soul,
How shall the feeble muse pursue through all
The vast extent of thy unbounded thought,
That even seeks th' unseen recesses dark
To penetrate of providence immense.
And thou the great dispenser of the world
Propitious, who with inspiration taught'st
Our greatest bard to send thy praises forth ;
Thou, who gav'st NEWTON thought ; who smil'dst serene,
When to its bounds he stretch'd his swelling soul ;
~~Who~~ still benignant ever blest his toil,
And deign'd to his enlight'ned mind t' appear
Confess'd around th' interminated world :
To me O thy divine infusion grant
(O thou in all so infinitely good)
'That I may sing thy everlasting works,
Thy inexhausted store of providence,
In thought effulgent and resounding verse.
O could I spread the wond'rous theme around,
Where the wind cools the oriental world,
To the calm breezes of the Zephir's breath,
To where the frozen hyperborean blasts,
To where the boist'rous tempest-leading south
From their deep hollow caves send forth their storms.
Thou still indulgent parent of mankind,
Lest humid emanations should no more
Flow from the ocean, but dissolve away
Through the long series of revolving time ;
And lest the vital principle decay,
By which the air supplies the springs of life ;
'Thou hast the fiery visag'd comets form'd

[b]

With

A POEM on Sir ISAAC NEWTON.

With vivifying spirits all replete,
Which they abundant breathe about the void,
Renewing the prolifick soul of things.
No longer now on thee amaz'd we call,
No longer tremble at imagin'd ills;
When comets blaze tremendous from on high,
Or when extending wide their flaming trains
With hideous grasp the skies engirdle round,
And spread the terrors of their burning locks.
For these through orbits in the length'ning space
Of many tedious rouling years compleat
Around the sun move regularly on ;
And with the planets in harmonious orbs,
And mystick periods their obeysance pay
To him majestick ruler of the skies
Upon his throne of circled glory fixt.
He or some god conspicuous to the view,
Or else the substitute of nature seems,
Guiding the courses of revolving worlds.
He taught great NEWTON the all-potent laws
Of gravitation, by whose simple power
The universe exists. Nor here the sage
Big with invention still renewing staid.
But O bright angel of the lamp of day,
How shall the muse display his greatest toil?
Let her plunge deep in Aganippe's waves,
Or in Castalia's ever-flowing stream,
That re-inspired she may sing to thee,
How NEWTON dar'd advent'rous to unbraid
The yellow tresses of thy shining hair.
Or didst thou gracious leave thy radiant sphere,
And to his hand thy lucid splendours give,
T' unweave the light-diffusing wreath, and part

The

A POEM on Sir ISAAC NEWTON.

The blended glories of thy golden plumes?
He with laborious, and unerring care,
How diff'rent and imbodied colours form
Thy piercing light, with just distinction found.
He with quick sight pursu'd thy darting rays,
When penetrating to th' obscure recess
Of solid matter, there perspicuous saw,
How in the texture of each body lay
The power that separates the diff'rent beams.
Hence over nature's unadorned face
Thy bright diversifying rays dilate
Their various hues: and hence when vernal rains
Descending swift have burst the low'ring clouds,
Thy splendors through the dissipating mists
In its fair vesture of unnumber'd hues
Array the show'ry bow. At thy approach
The morning risen from her pearly couch
With rosy blushes decks her virgin cheek;
The ev'ning on the frontispiece of heav'n
His mantle spreads with many colours gay;
The mid-day skies in radiant azure clad,
The shining clouds, and silver vapours rob'd
In white transparent intermixt with gold,
With bright variety of splendor cloath
All the illuminated face above.
When hoary-headed winter back retires
To the chill'd-pole, there solitary sits
Encompas'd round with winds and tempests bleak
In caverns of impenetrable ice,
And from behind the dissipated gloom
Like a new Venus from the parting fudge
The gay-apparell'd spring advances on;
When thou in thy meridian brightness sitt'st,
And from thy throne pure emanations flow

A POEM ON SIR ISAAC NEWTON.

Of glory bursting o'er the radiant skies :
Then let the muse Olympus' top ascend,
And o'er Thessalia's plain extend her view,
And count, O Tempe, all thy beauties o'er.
Mountains, whose summits grasp the pendant clouds,
Between their wood-invelop'd slopes embrace
The green-attired vallies. Every flow'r
Here in the pride of bounteous nature clad
Smiles on the bosom of th' enamell'd meads.
Over the smiling lawn the silver floods
Of fair Peneus gently roul along,
While the reflected colours from the flow'rs,
And verdant borders pierce the lympid waves,
And paint with all their variegated hue
The yellow sands beneath. Smooth gliding on
The waters hasten to the neighbouring sea.
Still the pleas'd eye the floating plain pursues;
At length, in Neptune's wide dominion lost,
Surveys the shining billows, that arise
Apparell'd each in Phœbus' bright attire :
Or from a far some tall majestic ship,
Or the long hostile lines of threat'ning fleets,
Which o'er the bright uneven mirror sweep,
In dazzling gold and waving purple deckt;
Such as of old, when haughty Athens power
Their hideous front, and terrible array
Against Pallene's coast extended wide,
And with tremendous war and battel stem
The trembling walls of Potidæa shook.
Crested with pendants curling with the breeze
The upright masts high bristle in the air,
Aloft exalting proud their gilded heads.
The silver waves against the painted prows
Raise their resplendent bosoms, and impearl

A POEM on Sir ISAAC NEWTON.

The fair vermilion with their glitt'ring drops :
And from on board the iron-leath'd hoft
Around the main a gleaming horrou'r caſts ;
Each flaming buckler like the mid-day ſun,
Each plumed helmet like the ſilver moon,
Each moving gauntlet like the light'ning's blaze,
And like a ſtar each brazen pointed ſpear.
But lo the ſacred high-erected fanes,
Fair citadels, and marble-crowned towers,
And ſumptuous palaces of ſtately towns
Magnificent ariſe, upon their heads
Bearing on high a wreath of ſilver light.
But ſee my muſe the high Pierian hill,
Behold its ſhaggy locks and airy top,
Up to the ſkies th' imperious mountain heaves .
The ſhining verdure of the nodding woods.
See where the ſilver Hippocrene flows,
Behold each glitt'ring rivulet, and rill
Through mazes wander down the green deſcent,
And ſparkle through the interwoven trees.
Here reſt a while and humble homage pay,
Here, where the ſacred genius, that inſpir'd
Sublime MÆONIDES and PINDAR's breaſt,
His habitation once was ſam'd to hold.
Here thou, O HOMER, offer'dſt up thy vows ;
Thee, the kind muſe CALLIOPEA heard,
And led thee to the empyrean ſeats,
There manifeſted to thy hallow'd eyes
The deeds of gods; thee wiſe MINERVA taught
The wondrous art of knowing human kind ;
Harmonious PHOEBUS tun'd thy heav'nly mind,
And ſwell'd to rapture each exalted ſenſe ;
Even MARS the dreadful battle-ruling god,
MARS taught thee war, and with his bloody hand

Inſtructed

A POEM on Sir ISAAC NEWTON.

Instructed thine, when in thy sounding lines
We hear the rattling of Bellona's carr,
The yell of discord, and the din of arms.
-PINDAR, when mounted on his fiery steed,
'Soars to the sun, opposing eagle like
His eyes undazzled to the fiercest rays.
He firmly seated, not like GLAUCUS' son,
Strides his swift-winged and fire-breathing horse,
And born aloft strikes with his ringing hoofs
The brazen vault of heav'n, superior there
Looks down upon the stars, whose radiant light
Illuminates innumerable worlds,
That through eternal orbits roul beneath.
But thou all hail immortalized son
Of harmony, all hail thou Thracian bard,
'To whom APOLLO gave his tuneful lyre.
O might'st thou, ORPHEUS, now again revive,
And NEWTON should inform thy list'ning ear
How the soft notes, and soul-enchancing strains
Of thy own lyre were on the wind convey'd.
He taught the muse, how sound progressive floats
Upon the waving particles of air,
When harmony in ever-pleasing strains,
Melodious melting at each lulling fall,
With soft alluring penetration steals
Through the enraptur'd ear to inmost thought,
And folds the senses in its silken bands.
So the sweet musick, which from ORPHEUS' touch
And fam'd AMPHION's, on the sounding string
Arose harmonious, gliding on the air,
Pierc'd the tough-bark'd and knotty-ribbed woods,
Into their saps soft inspiration breath'd
And taught attention to the stubborn oak.
Thus when great HENRY, and brave MARLB'ROUGH led

Th'

A POEM on Sir ISAAC NEWTON.

Th' imbattled numbers of BRITANNIA's sons,
The trump, that swells th' expanded cheek of fame,
That adds new vigour to the gen'rous youth,
And rouses sluggish cowardize it self,
The trumpet with its Mars-inciting voice,
The winds broad breast impetuous sweeping o'er
Fill'd the big note of war. Th' inspired host
With new-born ardor preſs the trembling GAUL ;
Nor greater throngs had reach'd eternal night,
Not if the fields of Agencourt had yawn'd
Expoſing horrible the gulf of fate ;
Or roaring Danube ſpread his arms abroad,
And overwhelm'd their legions with his floods.
But ~~as~~ the wand'ring muſe at length return ;
Nor yet, angelick genius of the ſun,
In worthy lays her high-attempting ſong
Has blazon'd forth thy venerated name.
Then let her ſweep the loud-reſounding lyre
Again, again o'er each melodious ſtring
Teach harmony to tremble with thy praiſe.
And ſtill thine ear O favourable grant,
And ſhe ſhall tell thee, that whatever charms,
Whatever beauties bloom on nature's face,
Procecd from thy all-influencing light.
That when ariſing with tempeſtuous rage,
The North impetuous rides upon the clouds
Diſperſing round the heav'ns obſtructive gloom,
And with his dreaded prohibition ſtays
The kind effuſion of thy genial beams ;
Pale are the rubies on AURORA's lips,
No more the roſes bluſh upon her cheeks,
Black are Peneus' ſtreams and golden ſands
In Tempe's vale dull melancholy ſits,
And every flower reclines its languid head.

By

A POEM on Sir ISAAC NEWTON.

By what high name shall I invoke thee, say,
Thou life-infusing deity, on thee
I call, and look propitious from on high,
While now to thee I offer up my prayer.
O had great NEWTON, as he found the cause,
By which found rouls thro' th' undulating air,
O had he, baffling times resistless power,
Discover'd what that subtle spirit is,
Or whatsoe'er diffusive else is spread
Over the wide-extended universe,
Which causes bodies to reflect the light,
And from their straight direction to divert
The rapid beams, that through their surface pierce.
But since embrac'd by th' icy arms of age,
And his quick thought by times cold hand congeal'd,
Ev'n NEWTON left unknown this hidden power ;
Thou from the race of human kind select
Some other worthy of an angel's care,
With inspiration animate his breast,
And him instruct in these thy secret laws.,
O let not NEWTON, to whose spacious view,
Now unobstructed, all th' extensive scenes
Of the ethereal ruler's works arise ;
When he beholds this earth he late adorn'd,
Let him not see philosophy in tears,
Like a fond mother solitary sit,
Lamenting him her dear, and only child.
But as the wife PYTHAGORAS, and he,
Whose birth with pride the fam'd Abdera boasts,
With expectation having long survey'd
This spot their antient seat, with joy beheld
Divine philosophy at length appear
In all her charms majestically fair,
Conducted by immortal NEWTON's hand :

A POEM on Sir ISAAC NEWTON.

So may he see another sage arise,
That shall maintain her empire: then no more
Imperious ignorance with haughty sway
Shall stalk rapacious o'er the ravag'd globe:
Then thou, O NEWTON, shalt protect these lines,
The humble tribute of the grateful muse;
Ne'er shall the sacrilegious hand despoil
Her laurel'd temples, whom his name preserves:
And were she equal to the mighty theme,
Futurity should wonder at her song;
Time should receive her with extended arms,
Seat her conspicuous in his rolling car,
And bear her down to his extremeſt bound.

FABLES with wonder tell how Terra's ſons
With iron force unloos'd the ſtubborn nerves
Of hills, and on the cloud-inſhrouded top
Of Pelion Oſſa pil'd. But if the vaſt
Gigantick deeds of ſavage ſtrength demand
Aſtoniſhment from men, what then ſhalt thou,
O what expreſſive rapture of the ſoul,
When thou before us, NEWTON, doſt diſplay
The labours of thy great excell'g mind;
When thou unveileſt all the wondrous ſcene,
The vaſt idea of th' eternal king,
Not dreadful bearing in his angry arm
The thunder hanging o'er our trembling heads;
But with th' effulgency of love replete,
And clad with power, which form'd th' extenſive heavens.
O happy he, whoſe enterprizing hand
Unbars the golden and relucid gates
Of th' empyrean dome, where thou enthron'd
Philoſophy art ſeated. Thou ſuſtain'd
By the firm hand of everlaſting truth

A POEM ON SIR ISAAC NEWTON.

Despiseſt all the injuries of time :
Thou never know'ſt decay when all around,
Antiquity obſcures her head. Behold
Th' Egyptian towers, the Babylonian walls,
And Thebes with all her hundred gates of braſs,
Behold them ſcatter'd like the duſt abroad.
Whatever now is flouriſhing and proud,
Whatever ſhall, muſt know devouring age.
Euphrates' ſtream, and ſeven-mouthed Nile,
And Danube, thou that from Germania's ſoil
To the black Euxine's far remoted ſhore,
O'er the wide bounds of mighty nations ſweep'ſt
In thunder loud thy rapid floods along.
Ev'n you ſhall feel inexorable time ;
To you the fatal day ſhall come ; no more
Your torrents then ſhall ſhake the trembling ground,
No longer then to inundations ſwol'n
Th' imperious waves the fertile paſtures drench,
But ſhrunk within a narrow channel glide ;
Or through the year's reiterated courſe
When time himſelf grows old, your wond'rous ſtreams
Loſt ev'n to memory ſhall lie unknown
Beneath obſcurity, and Chaos whelm'd.
But ſtill thou ſun illuminateſt all
The azure regions round, thou guideſt ſtill
The orbits of the planetary ſpheres ;
The moon ſtill wanders o'er her changing courſe,
And ſtill, O NEWTON, ſhall thy name ſurvive :
As long as nature's hand directs the world,
When ev'ry dark obſtruction ſhall retire,
And ev'ry ſecret yield its hidden ſtore,
Which thee dim-ſighted age forbade to ſee
Age that alone could ſtay thy riſing ſoul.
And could mankind among the fixed ſtars,

E'en

A POEM on Sir ISAAC NEWTON,

E'en to th' extremest bounds of knowledge reach,
To those unknown innumerable suns,
Whose light but glimmers from those distant worlds,
Ev'n to those utmost boundaries, those bars
That shut the entrance of th' illumin'd space
Where angels only tread the vast unknown,
Thou ever should'st be seen immortal there :
In each new sphere, each new-appearing sun,
In farthest regions at the very verge
Of the wide universe should'st thou be seen.
And lo, th' all-potent goddess NATURE takes
With her own hand thy great, thy just reward
Of immortality ; aloft in air
She displays, and with eternal grasp
Uprears the trophies of great NEWTON's fame.

R. GLOVER.

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ERRATA.

PAGE 25. line 4. read *In these Precepts*. p. 40. l. 24. for *I* read *X*. p. 53. l. penult. f. *E*. r. *F*. p. 82. l. ult. f. 40. r. 42. p. 83. l. ult. f. 43. r. 45. p. 91. l. 3. f. 48. r. 50. ibid. l. 25. for 49. r. 51. p. 92. l. 18. f. *AGFE*. r. *ENGFC*. p. 96. l. 23. dele the comma after $\frac{1}{2}$. p. 140. l. 12. dele *and*. p. 144. l. 15. f. *threefold*. r. *twofold*. p. 162. l. 25. f. $\frac{1}{2}$. r. $\frac{1}{4}$. p. 193. l. 2. r. *always*. p. 199. l. penult. and p. 200. l. 3. f. *F*. r. *C*. p. 201. l. 8. f. *ascends*. r. *must ascend*. ibid. l. 10. f. *it descends*. r. *descend*. p. 208. l. 14. f. *WTO*. r. *NTO*. In fig. 110. draw a line from *I* through *T*, till it meets the circle *ADCB*, where place *W*. p. 216. l. penult. f. *actum*. r. *maison*. p. 221. l. 23. f. *AF*. r. *AH*. p. 232. l. 23. after *invention* put a full point. p. 253. l. penult. dele the comma after *remarkable*. p. 255. l. ult. f. *DE*. r. *BE*. p. 278. l. 17. f. $\xi\tau$. r. $\xi\pi$. p. 299. l. 19. r. *the*. p. 361. l. 12. f. *I*. r. *t*. p. 369. l. 23. r. *Pseudo-topaz*. p. 378. l. 12. f. *thus*. r. *than*. p. 379. l. 15. f. *converge*. r. *diverge*. p. 384. l. 7. f. *optic glass*. r. *optic nerve*. p. 391. l. 18. r. *as 50 to 78*. p. 392. l. 18. after *telescope* add *be about 100 feet long and the*. in fig. 161. f. *A* put *s*. p. 399. l. 8. r. *AN*, *AN*. &c. p. 400. l. 19. r. *A π* , *A ρ* , *A τ* , *A ϕ* . p. 401. l. 14. r. fig. 163. The pages 374, 375, 376 are erroneously numbered 375, 376, 377; and the pages 382, 383 are numbered 381, 382.

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INTRODUCTION.



THE manner, in which Sir ISAAC NEWTON has published his philosophical discoveries, occasions them to lie very much concealed from all, who have not made the mathematics particularly their study. He once, indeed, intended to deliver, in a more familiar way, that part of his inventions, which relates to the system of the world ; but upon farther consideration he altered his design. For as the nature of those discoveries made it impossible to prove them upon any other than geometrical principles ; he apprehended, that those, who should not fully perceive the force of his arguments, would hardly be prevailed on to exchange their former sentiments for new opinions, so very different from

what were commonly received ^a. He therefore chose rather to explain himself only to mathematical readers; and declined the attempting to instruct such in any of his principles, who, by not comprehending his method of reasoning, could not, at the first appearance of his discoveries, have been persuaded of their truth. But now, since Sir ISAAC NEWTON's doctrine has been fully established by the unanimous approbation of all, who are qualified to understand the same; it is without doubt to be wished, that the whole of his improvements in philosophy might be universally known. For this purpose therefore I drew up the following papers, to give a general notion of our great philosopher's inventions to such, as are not prepared to read his own works, and yet might desire to be informed of the progress, he has made in natural knowledge; not doubting but there were many, besides those, whose turn of mind had led them into a course of mathematical studies, that would take great pleasure in tasting of this delightful fountain of science.

2. IT is a just remark, which has been made upon the human mind, that nothing is more suitable to it, than the contemplation of truth; and that all men are moved with a strong desire after knowledge; esteeming it honourable to excel therein; and holding it, on the contrary, disgraceful to mistake, err, or be in any way deceived. And this sentiment is by nothing more fully illustrated, than by the inclination of men to gain an acquaintance with the operations of nature; which disposition to enquire after the causes of things is

^a Philosoph. Nat. princ. math. L. iii. introduct.

so general, that all men of letters, I believe, find themselves influenced by it. Nor is it difficult to assign a reason for this, if we consider only, that our desire after knowledge is an effect of ~~that~~ taste for the sublime and the beautiful in things, which chiefly constitutes the difference between the human life, and the life of brutes. These inferior animals partake with us of the pleasures, that immediately flow from the bodily senses and appetites; but our minds are furnished with a superior sense, by which we are capable of receiving various degrees of delight, where the creatures below us perceive no difference. Hence arises that pursuit of grace and elegance in our thoughts and actions, and in all things belonging to us, which principally creates employment for the active mind of man. The thoughts of the human mind are too extensive to be confined only to the providing and enjoying of what is necessary for the support of our being. It is this taste, which has given rise to poetry, oratory, and every branch of literature and science. From hence we feel great pleasure in conceiving strongly, and in apprehending clearly, even where the passions are not concerned. Perspicuous reasoning appears not only beautiful; but, when set forth in its full strength and dignity, it partakes of the sublime, and not only pleases, but warms and elevates the soul. This is the source of our strong desire of knowledge; and the same taste for the sublime and the beautiful directs us to chuse particularly the productions of nature for the subject of our contemplation: our creator having so adapted our minds to the condition, wherein he has placed us, that all his visible

works, before we inquire into their make, strike us with the most lively ideas of beauty and magnificence.

. 3. BUT if there be so strong a passion in contemplative minds for natural philosophy; all such must certainly receive a particular pleasure in being informed of Sir ISAAC NEWTON'S discoveries, who alone has been able to make any great advancements in the true course leading to natural knowledge: whereas this important subject had before been usually attempted with that negligence, as cannot be reflected on without surprize. Excepting a very few, who, by pursuing a more rational method, had gained a little true knowledge in some particular parts of nature; the writers in this science had generally treated of it after such a manner, as if they thought, that no degree of certainty was ever to be hoped for. The custom was to frame conjectures; and if upon comparing them with things, there appeared some kind of agreement, though very imperfect, it was held sufficient. Yet at the same time nothing less was undertaken than intricate systems, and fathoming at once the greatest depths of nature; as if the secret causes of natural effects, contrived and framed by infinite wisdom, could be searched out by the slightest endeavours of our weak understandings. Whereas the only method, that can afford us any prospect of success in this difficult work, is to make our enquiries with the utmost caution, and by very slow degrees. And after our most diligent labour, the greatest part of nature will, no doubt, for ever remain beyond our reach.

I N T R O D U C T I O N.

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4. THIS neglect of the proper means to enlarge our knowledge, joined with the presumption to attempt, what was quite out of the power of our limited faculties, the Lord BACON judiciously observes to be the great obstruction to the progress of Science^a. Indeed that excellent person was the first, who expressly writ against this way of philosophizing; and he has laid open at large the absurdity of it in his admirable treatise, intitled NOVUM ORGANON SCIENTIARUM; and has there likewise described the true method, which ought to be followed.

5. THERE are, saith he, but two methods, that can be taken in the pursuit of natural knowledge. One is to make a hasty transition from our first and slight observations on things to general axioms, and then to proceed upon those axioms, as certain and uncontestable principles, without farther examination. The other method; (which he observes to be the only true one, but to his time unattempted;) is to proceed cautiously, to advance step by step, reserving the most general principles for the last result of our inquiries^b. Concerning the first of these two methods; where objections, which happen to appear against any such axioms taken up in haste, are evaded by some frivolous distinction, when the axiom it self ought rather to be corrected^c; he affirms, that the united endeavours of all ages cannot make it successful; because this original error in the first digestion of the mind (as he expresses himself) cannot afterwards be remedied^d: whereby he would signify to us, that if we set out in a

^a Nov. Org. Scient. L. i. Aphorism. 9.

^b Nov. Org. L. i. Aph. 19

^c Ibid. Aph. 25.

^d Aph. 30. Erroris radicales & in prima digestionis mentis ab excellentia functionum & remedium sequentium non curantur.

wrong

6 INTRODUCTION.

wrong way ; no diligence or art, we can use, while we follow so erroneous a course, will ever bring us to our designed end. And doubtless it cannot prove otherwise ; for in this spacious field of nature, if once we forsake the true path, we shall immediately lose our selves, and must for ever wander with uncertainty.

6. THE impossibility of succeeding in so faulty a method of philosophizing his Lordship endeavours to prove from the many false notions and prejudices, to which the mind of man is exposed ^a. And since this judicious writer apprehends, that men are so exceeding liable to fall into these wrong tracks of thinking, as to incur great danger of being misled by them, even while they enter on the true course in pursuit of nature ^b ; I trust, I shall be excused, if, by insisting a little particularly upon this argument, I endeavour to remove whatever prejudice of this kind, might possibly entangle the mind of any of my readers.

7. HIS Lordship has reduced these prejudices and false modes of conception under four distinct heads ^c.

8. THE first head contains such, as we are subject to from the very condition of humanity ; through the weakness both of our senses, and of the faculties of the mind ^d ; seeing, as this author well observes, the subtilty of nature far exceeds the greatest subtilty of our senses or acuteſt reasonings ^e. One

^a Aph. 38.

^b Ibid.

^c Aph. 39.

^d Aph. 41.

^e Aph. 10, 24.

of the false modes of conception, which he mentions under this head, is the forming to our selves a fanciful simplicity and regularity in natural things. This he illustrates by the following instances; the conceiving the planets to move in perfect circles; the adding an orb of fire to the other three elements, and the supposing each of these to exceed the other in rarity, just in a decuple proportion ^a. And of the same nature is the assertion of DES CARTES, without any proof, that all things are made up of three kinds of matter only ^b. As also this opinion of another philosopher; that light, in passing through different mediums, was refracted, so as to proceed by that way, through which it would move more speedily, than through any other ^c. The second erroneous turn of mind, taken notice of by his Lordship under this head, is, that all men are in some degree prone to a fondness for any notions, which they have once imbibed; whereby they often wrest things to reconcile them to those notions, and neglect the consideration of whatever will not be brought to an agreement with them; just as those do, who are addicted to judicial astrology, to the observation of dreams, and to such-like superstitions; who carefully preserve the memory of every incident, which serves to confirm their prejudices, and let slip out of their minds all instances, that make against them ^d. There is also a farther impediment to true knowledge, mentioned under the same head by this noble writer, which is; that whereas, through the weakness and imperfection of our senses, many things are concealed

^a Aph 45.

^b Des Cartes Princ. Phil. Part. 3. §. 52.

^c Fermat, in Oper. pag 156, &c.

^d Nov. Org. Aph. 46.

from

from us, which have the greatest effect in producing natural appearances; our minds are ordinarily most affected by that, which makes the strongest impression on our organs of sense; whereby we are apt to judge of the real importance of things in nature by a wrong measure^a. So, because the figuration and the motion of bodies strike our senses more immediately than most of their other properties, DES CARTES and his followers will not allow any other explication of natural appearances, than from the figure and motion of the parts of matter. By which example we see how justly his Lordship observes this cause of error to be the greatest of any^b; since it has given rise to a fundamental principle in a system of philosophy, that not long ago obtained almost an universal reputation.

9. THESE are the chief branches of those obstructions to knowledge, which this author has reduced under his first head of false conceptions. The second head contains the errors, to which particular persons are more especially obnoxious^c. One of these is the consequence of a preceding observation: that as we are exposed to be captivated by any opinions, which have once taken possession of our minds; so in particular, natural knowledge has been much corrupted by the strong attachment of men to some one part of science, of which they reputed themselves the inventors, or about which they have spent much of their time; and hence have been apt to conceive it to be of greater use in the study of na-

^a Aph. 50.
^b Ibid.

^c Aph. 53.

tural philosophy than it was : like ARISTOTLE, who reduced his physics to logical disputations; and the chymists, who thought, that nature could be laid open only by the force of their fires ^a. Some again are wholly carried away by an excessive veneration for antiquity; others, by too great fondness for the moderns; few having their minds so well balanced, as neither to depreciate the merit of the ancients, nor yet to despise the real improvements of later times ^b. To this is added by his Lordship a difference in the genius of men, that some are most fitted to observe the similitude, there is in things, while others are more qualified to discern the particulars, wherein they disagree; both which dispositions of mind are useful: but to the prejudice of philosophy men are apt to run into excess in each; while one sort of genius dwells too much upon the gross and sum of things, and the other upon trifling minuteneffes and shadowy distinctions ^c.

~~to~~ UNDER the third head of prejudices and false notions this writer considers such, as follow from the lax and indefinite use of words in ordinary discourse; which occasions great ambiguities and uncertainties in philosophical debates (as another eminent philosopher has since shewn more at large ^d;) inso-much that this our author thinks a strict defining of terms to be scarce an infallible remedy against this inconvenience ^e. And perhaps he has no small reason on his side: for the common inaccurate sense of words, notwithstanding the limitations given them by definitions, will offer it self so constantly to

^a Aph. 54.

^b Aph. 56.

^c Aph. 55.

^d Locke, On human understanding, B. iii

^e Nov. Org. Aph. 59.

the mind, as to require great caution and circumspection for us not to be deceived thereby. Of this we have a very eminent instance in the great disputes, that have been raised about the use of the word attraction in philosophy; of which ~~we~~ shall be obliged hereafter to make particular mention^a. Words thus to be guarded against are of two kinds. Some are names of things, that are only imaginary^b; such words are wholly to be rejected. But there are other terms, that allude to what is real, though their signification is confused^c. And these latter must of necessity be continued in use; but their sense cleared up, and freed, as much as possible, from obscurity.

II. THE last general head of these errors comprehends such, as follow from the various sects of false philosophies; which this author divides into three sorts, the sophistical, empirical, and superstitious^d. By the first of these he means a philosophy built upon speculations only without ~~experi-~~ments^e; by the second, where experiments are blindly adhered to, without proper reasoning upon them^f; and by the third, wrong opinions of nature fixed in mens minds either through false religions, or from misunderstanding the declarations of the true^g.

12. THESE are the four principal canals, by which this judicious author thinks, that philosophical errors have flowed in upon us. And he rightly observes, that the faulty method of

^a In the conclusion.

^b Nov. Org. L. i. Aph. 59.

^c Ibid. Aph. 60.

^d Ibid. Aph. 62.

^e Aph. 63.

^f Aph. 64.

^g Aph. 65.

proceeding

proceeding in philosophy, against which he writes ^a, is so far from assisting us towards overcoming these prejudices ; that he apprehends it rather suited to rivet them more firmly to the mind ^b. How great reason then has his Lordship to call this way of philosophizing the parent of error, and the bane of all knowledge ^c ? For, indeed, what else but mistakes can so bold and presumptuous a treatment of nature produce ? have we the wisdom necessary to frame a world, that we should think so easily, and with so slight a search to enter into the most secret springs of nature, and discover the original causes of things ? what chimeras, what monsters has not this preposterous method brought forth ? what schemes, or what hypothesis's of the subtlest wits has not a stricter enquiry into nature not only overthrown, but manifested to be ridiculous and absurd ? Every new improvement, which we make in this science, lets us see more and more the weakness of our guesses. Dr. HARVEY, by that one discovery of the circulation of the blood, has dissipated all the speculations and reasonings of many ages upon the animal oeconomy. A SELLIUS, by detecting the lacteal veins, shewed how little ground all physicians and philosophers had in conjecturing, that the nutritive part of the aliment was absorbed by the mouths of the veins spread upon the bowels : and then PECQUET, by finding out the thoracic duct, as evidently proved the vanity of the opinion, which was persisted in after the lacteal vessels were known, that the alimental juice was conveyed immediately to the liver, and ~~there converted~~ into blood.

^a See above, § 4, 5.
^b Nov. Org. L. i. Aph. 69

^c Ibid.

13. As these things set forth the great absurdity of proceeding in philosophy on conjectures, by informing us how far the operations of nature are above our low conceptions; so on the other hand, such instances of success from a more judicious method shew us, that our bountiful maker has not left us wholly without means of delighting our selves in the contemplation of his wisdom. That by a just way of inquiry into nature, we could not fail of arriving at discoveries very remote from our apprehensions; the Lord BACON himself argues from the experience of mankind. If, says he, the force of guns should be described to any one ignorant of them, by their effects only; he might reasonably suppose, that those engines of destruction were only a more artificial composition, than he knew, of wheels and other mechanical powers: but it could never enter his thoughts, that their immense force should be owing to a peculiar substance, which would enkindle into so violent an explosion, as we experience in gunpowder: since he would nowhere see the least example of any such operation; except perhaps in earthquakes and thunder, which he would doubtless look upon as exalted powers of nature, greatly surpassing any art of man to imitate. In the same manner, if a stranger to the original of silk were shewn a garment made of it, he would be very far from imagining so strong a substance to be spun out of the bowels of a small worm; but must certainly believe it either a vegetable substance, like flax or cotton; or the natural covering of some animal, as wool is of sheep. Or had we been told, before the invention of the magnetic needle among us, that another people was in possession of a certain contrivance

contrivance, by which they were inabled to discover the position of the heavens, with vastly more ease, than we could do ; what could have been imagined more, than that they were provided with some fitter astronomical instrument for this purpose than we ? That any stone should have so amazing a property, as we find in the magnet, must have been the remotest from our thoughts¹.

14. BUT what surprizing advancements in the knowledge of nature may be made by pursuing the true course in philosophical inquiries ; when those searches are conducted by a genius equal to so divine a work, will be best understood by considering Sir ISAAC NEWTON's discoveries. That my reader may apprehend as just a notion of these, as can be conveyed to him, by the brief account, which I intend to lay before him ; I have set apart this introduction for explaining, in the fullest manner I am able, the principles, whereon Sir ISAAC NEWTON proceeds. For without a clear conception of these, it is impossible to form any true idea of the singular excellence of the inventions of this great philosopher.

15. THE principles then of this philosophy are ; upon no consideration to indulge conjectures concerning the powers and laws of nature, but to make it our endeavour with all diligence to search out the real and true laws, by which the constitution of things is regulated. The philosopher's first care must be to distinguish, what he sees to be within his power, from what

¹ Ibid. Aph. 109

is beyond his reach ; to assume no greater degree of knowledge, than what he finds himself possessed of ; but to advance by slow and cautious steps ; to search gradually into natural causes ; to secure to himself the knowledge of the most immediate cause of each appearance, before he extends his views farther to causes more remote. This is the method, in which philosophy ought to be cultivated ; which does not pretend to so great things, as the more airy speculations ; but will perform abundantly more : we shall not perhaps seem to the unskilful to know so much, but our real knowledge will be greater. And certainly it is no objection against this method, that some others promise, what is nearer to the extent of our wishes : since this, if it will not teach us all we could desire to be informed of, will however give us some true light into nature ; which no other can do. Nor has the philosopher any reason to think his labour lost, when he finds himself stopt at the cause first discovered by him, or at any other more remote cause, ~~short~~ of the original : for if he has, but sufficiently proved any one cause, he has entered so far into the real constitution of things, has laid a safe foundation for others to work upon, and has facilitated their endeavours in the search after yet more distant causes ; and besides, in the mean time he may apply the knowledge of these intermediate causes to many useful purposes. Indeed the being able to make ~~practical~~ deductions from natural causes, constitutes the great distinction between the true philosophy and the false. Causes assumed upon conjecture, must be so loose and undefined, that nothing particular can be collected from them. But those causes, which are brought to light by a strict examination of

of things, will be more distinct. Hence it appears to have been no unuseful discovery, that the ascent of water in pumps is owing to the pressure of the air by its weight or spring; though the causes, which make the air gravitate, and render it elastic, be unknown: for notwithstanding we are ignorant of the original, whence these powers of the air are derived; yet we may receive much advantage from the bare knowledge of these powers. If we are but certain of the degree of force, wherewith they act, we shall know the extent of what is to be expected from them; we shall know the greatest height, to which it is possible by pumps to raise water; and shall thereby be prevented from making any useless efforts towards improving these instruments beyond the limits prescribed to them by nature; whereas without so much knowledge as this, we might probably have wasted in attempts of this kind much time and labour. How long did philosophers busy themselves to no purpose in endeavouring to perfect telescopes, by forming the glasses into some new figure; till Sir ISAAC NEWTON demonstrated, that the effects of telescopes were limited from another cause, than was supposed; which no alteration in the figure of the glasses could remedy? What method Sir ISAAC NEWTON himself has found for the improvement of telescopes shall be explained hereafter^a. But at present I shall proceed to illustrate, by some farther instances, this distinguishing character of the true philosophy, which we have now under consideration. It was no trifling discovery, ~~that the contraction of the muscles of animals puts their~~ limbs in motion, though the original cause of that contraction

^a Book III. Chap. iv.

remains

remains a secret, and perhaps may always do so; for the knowledge of thus much only has given rise to many speculations upon the force and artificial disposition of the muscles, and has opened no narrow prospect into the animal fabrick. The finding out, that the nerves are great agents in this action, leads us yet nearer to the original cause, and yields us a wider view of the subject. And each of these steps affords us assistance towards restoring this animal motion, when impaired in our selves, by pointing out the seats of the injuries, to which it is obnoxious. To neglect all this, because we can hitherto advance no farther, is plainly ridiculous. It is confessed by all, that GALILEO greatly improved philosophy, by shewing, as we shall relate hereafter, that the power in bodies, which we call gravity, occasions them to move downwards with a velocity equally accelerated^a; and that when any body is thrown forwards, the same power obliges it to describe in its motion that line, which is called by geometers a parabola^b: yet we are ignorant of the cause, which makes bodies gravitate. But although we are unacquainted with the spring, whence this power in nature is derived, nevertheless we can estimate its effects. When a body falls perpendicularly, it is known, how long time it takes in descending from any height whatever: and if it be thrown forwards, we know the real path, which it describes; we can determine in what direction, and with what degree of swiftness it must be projected, in order to its striking against any object desired; and we can also ascertain the very force, wherewith it will strike

^a Book I. Chap. 2. § 14.^b Ibid § 85. &c.

Sir ISAAC NEWTON has farther taught, that this power of gravitation extends up to the moon, and causes that planet to gravitate as much towards the earth, as any of the bodies, which are familiar to us, would, if placed at the same distance^a: he has proved likewise, that all the planets gravitate towards the sun, and towards one another; and that their respective motions follow from this gravitation. All this he has demonstrated upon indisputable geometrical principles, which cannot be rendered precarious for want of knowing what it is, which causes these bodies thus mutually to gravitate: any more than we can doubt of the propensity in all the bodies about us, to descend towards the earth; or can call in question the fore-mentioned propositions of GALILEO, which are built upon that principle. And as GALILEO has shewn more fully, than was known before, what effects were produced in the motion of bodies by their gravitation towards the earth; so Sir ISAAC NEWTON, by this his invention, has much advanced our knowledge in the celestial motions. By discovering that the moon gravitates towards the sun, as well as towards the earth; he has laid open those intricacies in the moon's motion, which no astronomer, from observations only, could ever find out^b: and one kind of heavenly bodies, the comets, have their motion now clearly ascertained; whereof we had before no true knowledge at all^c.

* 16. DOUBTLESS it might be expected, that such surprizing success should have silenced, at once, every cavil. But we

^a See Book II. Ch. 3 § 3. 4. of this treatise.
^b See Book II. Ch. 3. of this treatise.

^c See Chap 4

have seen the contrary. For because this philosophy professes modestly to keep within the extent of our faculties, and is ready to confess its imperfections, rather than to make any fruitless attempts to conceal them, by seeking to cover the defects in our knowledge with the vain ostentation of rash and groundless conjectures; hence has been taken an occasion to insinuate that we are led to miraculous causes, and the occult qualities of the schools.

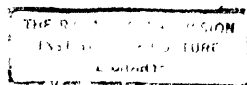
17. BUT the first of these accusations is very extraordinary. If by calling these causes miraculous nothing more is meant than only, that they often appear to us wonderful and surprizing, it is not easy to see what difficulty can be raised from thence; for the works of nature discover every where such proofs of the unbounded power, and the consummate wisdom of their author, that the more they are known, the more they will excite our admiration: and it is too manifest to be insisted on, that the common sense of the word ~~miraculous~~ can have no place here, when it implies what is above the ordinary course of things. The other imputation, that these causes are occult upon the account of our not perceiving what produces them, contains in it great ambiguity. That something relating to them lies hid, the followers of this philosophy are ready to acknowledge, nay desire it should be carefully remarked, as pointing out proper subjects for future inquiry. But this is very different from the proceeding of the schoolmen in the causes called by them occult. For as their occult qualities were understood to operate ~~in a manner~~ occult, and not apprehended by us; so they were obtruded.

~~added~~ upon us for such original and essential properties in bodies, as made it vain to seek any farther cause; and a greater power was attributed to them, than any natural appearances authorized. For instance, the rise of water in pumps was ascribed to a certain abhorrence of a vacuum, which they thought fit to assign to nature. And this was so far a true observation, that the water does move, contrary to its usual course, into the space, which otherwise would be left void of any sensible matter; and, that the procuring such a vacuity was the apparent cause of the water's ascent. But while we were not in the least informed how this power, called an abhorrence of a vacuum, produced the visible effects; instead of making any advancement in the knowledge of nature, we only gave an artificial name to one of her operations: and when the speculation was pushed so beyond what any appearances required, as to have it concluded, that this abhorrence of a vacuum was a power inherent in all matter, and so unlimited as to render it impossible for a vacuum to exist at all; it then became a much greater absurdity, in being made the foundation of a most ridiculous manner of reasoning; as at length evidently appeared, when it came to be discovered, that this rise of the water followed only from the pressure of the air, and extended it self no farther, than the power of that cause. The scholastic stile in discoursing of these occult qualities, as if they ~~were~~ essential differences in the very substances, of which bodies consisted, was certainly very absurd; by reason it tended to discourage all farther inquiry. But no such ill consequences can follow from the considering of any natural causes, which confessedly are not traced up to

their first original. How shall we ever come to the knowledge of the several original causes of things, otherwise than by storing up all intermediate causes which we can discover? Are all the original and essential properties of matter so very obvious, that none of them can escape our first view? This is not probable. It is much more likely, that, if some of the essential properties are discovered by our first observations, a stricter examination should bring more to light.

18. BUT in order to clear up this point concerning the essential properties of matter, let us consider the subject a little distinctly. We are to conceive, that the matter, out of which the universe of things is formed, is furnished with certain qualities and powers, whereby it is rendered fit to answer the purposes, for which it was created. But every property, of which any particle of this matter is in it self possessed, and which is not barely the consequence of the union of this particle with other portions of matter, we may call an essential property: whereas all other qualities or attributes belonging to bodies, which depend on their particular frame and composition, are not essential to the matter, whereof such bodies are made; because the matter of these bodies will be deprived of those qualities, only by the dissolution of the body, without working any change in the original constitution of one single particle of this mass of matter. Extension we apprehend to be one of these essential properties, and impenetrability another. These two belong ~~universally to all matter~~; and are the principal ingredients in the idea, which this word matter usually excites in the mind. Yet as the idea, marked
by

By this name, is not purely the creature of our own understandings, but is taken for the representation of a certain substance without us ; if we should discover, that every part of the substance, in which we find these two properties, should likewise be endowed universally with any other essential qualities ; all these, from the time they come to our notice, must be united under our general idea of matter. How many such properties there are actually in all matter we know not ; those, of which we are at present apprized, have been found out only by our observations on things ; how many more a farther search may bring to light, no one can say ; nor are we certain, that we are provided with sufficient methods of perception to discern them all. Therefore, since we have no other way of making discoveries in nature, but by gradual inquiries into the properties of bodies ; our first step must be to admit without distinction all the properties, which we observe ; and afterwards we must endeavour, as far as we are able, to distinguish between the qualities, wherewith the very substances themselves are indued, and those appearances, which result from the structure only of compound bodies. Some of the properties, which we observe in things, are the attributes of particular bodies only ; others universally belong to all, that fall under our notice. Whether some of the qualities and powers of particular bodies, be derived from different kinds of matter entering their composition, cannot, in the present imperfect state of our knowledge, absolutely be decided ; though we have not yet any reason to conclude, but that all the bodies, with which we converse, are framed out of the very same kind of matter, and that their distinct quali-



qualities are occasioned only by their structure; through the variety whereof the general powers of matter are caused to produce different effects. On the other hand, we should not hastily conclude, that whatever is found to appertain to all matter, which falls under our examination, must for that reason only be an essential property thereof, and not be derived from some unseen disposition in the frame of nature. Sir ISAAC NEWTON has found reason to conclude, that gravity is a property universally belonging to all the perceptible bodies in the universe, and to every particle of matter, whereof they are composed. But yet he no where asserts this property to be essential to matter. And he was so far from having any design of establishing it as such, that, on the contrary, he has given some hints worthy of himself at a cause for it^a; and expressly says, that he proposed those hints to shew, that he had no such intention^b.

19. It appears from hence, that it is not easy to determine, what properties of bodies are essentially inherent in the matter, out of which they are made, and what depend upon their frame and composition. But certainly whatever properties are found to belong either to any particular systems of matter, or universally to all, must be considered in philosophy; because philosophy will be otherwise imperfect. Whether those properties can be deduced from some other appertaining to matter, either among those, which are already known, or among such as can be discovered by us, is afterwards to be sought for the farther improvement of our knowledge. But this

^a At the end of his Optics.
in Qu. 21.

^b See the same treatise, in
Advertisment 2.

inquiry cannot properly have place in the deliberation about admitting any property of matter or bodies into philosophy; for that purpose it is only to be considered, whether the existence of such a property has been justly proved or not. Therefore to decide what causes of things are rightly received into natural philosophy, requires only a distinct and clear conception of what kind of reasoning is to be allowed of as convincing, when we argue upon the works of nature.

20. THE proofs in natural philosophy cannot be so absolutely conclusive, as in the mathematics. For the subjects of that science are purely the ideas of our own minds. They may be represented to our senses by material objects, but they are themselves the arbitrary productions of our own thoughts; so that as the mind can have a full and adequate knowledge of its own ideas, the reasoning in geometry can be rendered perfect. But in natural knowledge the subject of our contemplation is without us, and not so completely to be known: therefore our method of arguing must fall a little short of absolute perfection. It is only here required to steer a just course between the conjectural method of proceeding, against which I have so largely spoke; and demanding so rigorous a proof, as will reduce all philosophy to mere scepticism, and exclude all prospect of making any progress in the knowledge of nature.

21. THE concessions, which are to be allowed in this science, are by Sir ISAAC NEWTON included under a very few simple precepts.

22. THE

22. THE first is, that more causes are not to be received into philosophy, than are sufficient to explain the appearances of nature. That this rule is approved of unanimously, is evident from those expressions so frequent among all philosophers, that nature does nothing in vain; and that a variety of means, where fewer would suffice, is needless. And certainly there is the highest reason for complying with this rule. For should we indulge the liberty of multiplying, without necessity, the causes of things, it would reduce all philosophy to mere uncertainty; since the only proof, which we can have, of the existence of a cause, is the necessity of it for producing known effects. Therefore where one cause is sufficient, if there really should in nature be two, which is in the last degree improbable, we can have no possible means of knowing it, and consequently ought not to take the liberty of imagining, that there are more than one.

23. THE second precept is the direct consequence of the first, that to like effects are to be ascribed the same causes. For instance, that respiration in men and in brutes is brought about by the same means; that bodies descend to the earth here in EUROPE, and in AMERICA from the same principle; that the light of a culinary fire, and of the sun have the same manner of production; that the reflection of light is effected in the earth, and in the planets by the same power; and the like.

24. THE third of these precepts has equally evident reason for it. It is only, that those qualities, which in the same body can neither be lessened nor increased, and which belong
to

to all bodies that are in our power to make trial upon, ought to be accounted the universal properties of all bodies whatever.

25. IN this precept is founded that method of arguing by induction, without which no progress could be made in natural philosophy. For as the qualities of bodies become known to us by experiments only ; we have no other way of finding the properties of such bodies, as are out of our reach to experiment upon, but by drawing conclusions from those which fall under our examination. The only caution here required is, that the observations and experiments, we argue upon, be numerous enough, and that due regard be paid to all objections, that occur, as the Lord BACON very judiciously directs^a. And this admonition is sufficiently complied with, when by virtue of this rule we ascribe impenetrability and extension to all bodies, though we have no sensible experiment, that affords a direct proof of any of the celestial bodies being impenetrable ; nor that the fixed stars are so much as extended. For the more perfect our instruments are, whereby we attempt to find their visible magnitude, the less they appear ; insomuch that all the sensible magnitude, which we observe in them, seems only to be an optical deception by the scattering of their light. However, I suppose no one will imagine they are without any magnitude, though their immense distance makes it undiscernable by us. After the same manner, if it can be proved, that all

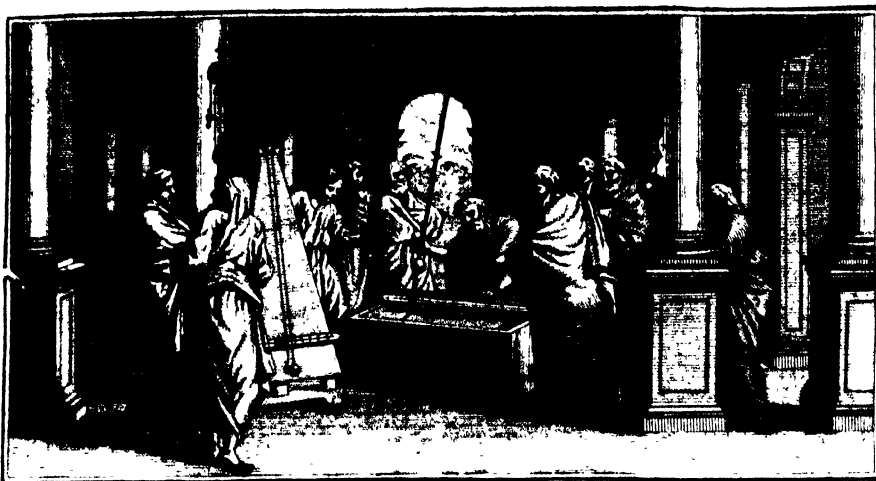
^a Nov. Org. Lib. i. Ax. 105.

bodies here gravitate towards the earth, in proportion to the quantity of solid matter in each ; and that the moon gravitates to the earth likewise, in proportion to the quantity of matter in it ; and that the sea gravitates towards the moon, and all the planets towards each other ; and that the very comets have the same gravitating faculty ; we shall have as great reason to conclude by this rule, that all bodies gravitate towards each other. For indeed this rule will more strongly hold in this case, than in that of the impenetrability of bodies ; because there will more instances be had of bodies gravitating, than of their being impenetrable.

25. THIS is that method of induction, whereon all philosophy is founded ; which our author farther inforces by this additional precept, that whatever is collected from this induction, ought to be received, notwithstanding any conjectural hypothesis to the contrary, till such times as it shall be contradicted or limited by farther observations on nature.



BOOK.



B O O K I.
 CONCERNING THE
MOTION of BODIES
 IN GENERAL.

CHAP. I.
 Of the LAWS of MOTION.



AVING thus explained Sir ISAAC
 NEWTON's method of reasoning in
 philosophy, I shall now proceed to
 my intended account of his discove-
 ries. These are contained in two trea-
 tises. In one of them, the MATHEMA-
 TICAL PRINCIPLES OF NATURAL PHILOSO-
 PHY, his chief design is to shew by what laws the heavenly

motions are regulated ; in the other, his OPTICS, he discoursed of the nature of light and colours, and of the action between light and bodies. This second treatise is wholly confined to the subject of light: except some conjectures proposed at the end concerning other parts of nature, which lie hitherto more concealed. In the other treatise our author was obliged to smooth the way to his principal intention, by explaining many things of a more general nature: for even some of the most simple properties of matter were scarce well established at that time. We may therefore reduce Sir ISAAC NEWTON's doctrine under three general heads ; and I shall accordingly divide my account into three books. In the first I shall speak of what he has delivered concerning the motion of bodies, without regard to any particular system of matter ; in the second I shall treat of the heavenly motions ; and the third shall be employed upon light.

2. IN the first part of my design, we must begin with an account of the general laws of motion.

3. THESE laws are some universal affections and properties of matter drawn from experience, which are made use of as axioms and evident principles in all our arguings upon the motion of bodies. For as it is the custom of geometers to assume in their demonstration some propositions, without exhibiting the proof of them ; so in philosophy, all our reasoning must be built upon some properties of matter, first admitted as principles whereon to argue. In geometry these axioms are thus assumed, on account of their being so evident

 as

as to make any proof in form needless. But in philosophy no properties of bodies can be in this manner received for self-evident; since it has been observed above, that we can conclude nothing concerning matter by any reasonings upon its nature and essence, but that we owe all the knowledge, we have thereof, to experience. Yet when our observations on matter have inform'd us of some of its properties, we may securely reason upon them in our farther inquiries into nature. And these laws of motion, of which I am here to speak, are found so universally to belong to bodies, that there is no motion known, which is not regulated by them. These are by Sir ISAAC NEWTON reduced to three ^a.

4. THE first law is, that all bodies have such an indifference to rest, or motion, that if once at rest they remain so, till disturbed by some power acting upon them: but if once put in motion, they persist in it; continuing to move right forwards perpetually, after the power, which gave the motion, is removed; and also preserving the same degree of velocity or quickness, as was first communicated, not stopping or remitting their course, till interrupted or otherwise disturbed by some new power impressed.

5. THE second law of motion is, that the alteration of the state of any body, whether from rest to motion, or from motion to rest, or from one degree of motion to another, is always proportional to the force impressed. A body at rest, when

^a Princip. philos. pag. 13, 14.

acted upon by any power, yields to that power, moving in the same line, in which the power applied is directed ; and moves with a less or greater degree of velocity, according to the degree of the power ; so that twice the power shall communicate a double velocity, and three times the power a threefold velocity. If the body be moving, and the power impressed act upon the body in the direction of its motion, the body shall receive an addition to its motion, as great as the motion, into which that power would have put it from a state of rest ; but if the power impressed upon a moving body act directly opposite to its former motion, that power shall then take away from the body's motion, as much as in the other case it would have added to it. Lastly, if the power be impressed obliquely, there will arise an oblique motion differing more or less from the former direction, according as the new impression is greater or less. For example, if the body A (in fig. I.) be moving in the direction AB, and when it is at the point A, a power be impressed upon it in the direction AC, the body shall from henceforth neither move in its first direction AB, nor in the direction of the adventitious power, but shall take a course as AD between them : and if the power last impressed be just equal to that, which first gave to the body its motion ; the line AD shall pass in the middle between AB and AC, dividing the angle under BAC into two equal parts ; but if the power last impressed be greater than the first, the line AD shall ~~incline~~ incline most to AC ; whereas if the last impression be less than the first, the line AD shall incline most to AB. To be more particular, the situation of
the

the line AD is always to be determined after this manner. Let AE be the space, which the body would have moved through in the line AB during any certain portion of time; provided that body, when at A, had received no second impulse. Suppose likewise, that AF is the part of the line AC, through which the body would have moved during an equal portion of time, if it had been at rest in A, when it received the impulse in the direction AC: then if from E be drawn a line parallel to, or equidistant from AC, and from F another line parallel to AB, those two lines will meet in the line AD.

6. THE third and last of these laws of motion is, that when any body acts upon another, the action of that body upon the other is equalled by the contrary reaction of that other body upon the first.

7. THESE laws of motion are abundantly confirmed by this, that all the deductions made from them, in relation to the motion of bodies, how complicated soever, are found to agree perfectly with observation. This shall be shewn more at large in the next chapter. But before we proceed to so diffusive a proof; I chuse here to point out those appearances of bodies, whereby the laws of motion are first suggested to us.

8. DAILY observation makes it appear to us, that any body, which we once see at rest, never puts it self into fresh motion;

motion; but continues always in the same place, till removed by some power applied to it.

9. A G A I N, whenever a body is once in motion, it continues in that motion some time after the moving power has quitted it, and it is left to it self. Now if the body continue to move but a single moment, after the moving power has left it, there can no reason be assigned, why it should ever stop without some external force. For it is plain, that this continuance of the motion is caused only by the body's having already moved, the sole operation of the power upon the body being the putting it in motion; therefore that motion continued will equally be the cause of its farther motion, and so on without end. The only doubt that can remain, is, whether this motion communicated continues intire, after the power, that caused it, ceases to act; or whether it does not gradually languish and decrease. And this suspicion cannot be removed by a transient and slight observation on bodies, but will be fully cleared up by those more accurate proofs of the laws of motion, which are to be considered in the next chapter.

10. L A S T L Y, bodies in motion appear to affect a straight course without any deviation, unless when disturbed by some adventitious power acting upon them. If a body be thrown perpendicularly upwards or downwards, it appears to continue in the same straight line during the whole time of its motion. If a body be thrown in any other direction, it is found to deviate from the line, in which it began to move, more and more

more continually towards the earth, whither it is directed by its weight : but since, when the weight of a body does not alter the direction of its motion, it always moves in a straight line, without doubt in this other case the body's declining from its first course is no more, than what is caused by its weight alone. As this appears at first sight to be unquestionable, so we shall have a very distinct proof thereof in the next chapter, where the oblique motion of bodies will be particularly considered.

II. THUS we see how the first of the laws of motion agrees with what appears to us in moving bodies. But here occurs this farther consideration, that the real and absolute motion of any body is not visible to us : for we are our selves also in constant motion along with the earth whereon we dwell ; insomuch that we perceive bodies to move so far only, as their motion is different from our own. When a body appears to us to lie at rest, in reality it only continues the motion, it has received, without putting forth any power to change that motion. If we throw a body in the course or direction, wherein we are carried our selves ; so much motion as we seem to have given to the body, so much we have truly added to the motion, it had, while it appeared to us to be at rest. But if we impel a body the contrary way, although the body appears to us to have received by such an impulse as much motion, as when impelled the other way ; yet in this case we have taken from the body so much real motion, as we seem to have given it. Thus the motion, which we see in bodies,

is not their real motion, but only relative with respect to us; and the forementioned observations only shew us, that this first law of motion has place in this relative or apparent motion. However, though we cannot make any observation immediately on the absolute motion of bodies, yet by reasoning upon what we observe in visible motion, we can discover the properties and effects of real motion.

12. WITH regard to this first law of motion, which is now under consideration, we may from the foregoing observations most truly collect, that bodies are disposed to continue in the absolute motion, which they have once received, without increasing or diminishing their velocity. When a body appears to us to lie at rest, it really preserves without change the motion, which it has in common with our selves: and when we put it into visible motion, and we see it continue that motion; this proves, that the body retains that degree of its absolute motion, into which it is put by our acting upon it: if we give it such an apparent motion, which adds to its real motion, it preserves that addition; and if our acting on the body takes off from its real motion, it continues afterwards to move with no more real motion, than we have left it.

13. AGAIN, we do not observe in bodies any disposition or power within themselves to change the direction of their motion; and if they had any such power, it would easily be discovered. For suppose a body by the structure or disposition of its parts, or by any other circumstance in its make, was induced

dued with a power of moving it self; this self-moving principle, which should be thus inherent in the body, and not depend on any thing external, must change the direction wherein it would act, as often as the position of the body was changed: so that for instance, if a body was lying before me in such a position, that the direction, wherein this principle disposes the body to move, was pointed directly from me; if I then gradually turned the body about, the direction of this self-moving principle would no longer be pointed directly from me, but would turn about along with the body. Now if any body, which appears to us at rest, were furnished with any such self-moving principle; from the body's appearing without motion we must conclude, that this self-moving principle lies directed the same way as the earth is carrying the body; and such a body might immediately be put into visible motion only by turning it about in any degree, that this self-moving principle might receive a different direction.

14. FROM these considerations it very plainly follows, that if a body were once absolutely at rest; not being furnished with any principle, whereby it could put it self into motion, it must for ever continue in the same place, till acted upon by something external: and also that when a body is put into motion, it has no power within it self to make any change in the direction of that motion; and consequently that the body must move on straight forward without declining any way whatever. But it has before been shewn, that bodies do not appear to have in themselves any power to

change the velocity of their motion: therefore this first law of motion has been illustrated and confirmed, as much as can be from the transient observations, which have here been discourf'd upon; and in the next chapter all this will be farther established by more correct observations.

15. BUT I shall now pass to the second law of motion; wherein, when it is asserted, that the velocity, with which any body is moved by the action of a power upon it, is proportional to that power; the degree of power is supposed to be measured by the greatness of the body, which it can move with a given celerity. So that the sense of this law is, that if any body were put into motion with that degree of swiftness, as to pass in one hour the length of a thousand yards; the power, which would give the same degree of velocity to a body twice as great, would give this lesser body twice the velocity, causing it to describe in the same space of an hour two thousand yards. But by a body twice as great as another, I do not here mean simply of twice the bulk, but one that contains a double quantity of solid matter.

16. WHY the power, which can move a body twice as great as another with the same degree of velocity, should be called twice as great as the power, which can give the lesser body the same velocity, is evident. For if we should suppose the greater body to be divided into two equal parts, each equal to the lesser body, each of these halves will require the same degree of power to move them with the velocity of the lesser body, as the lesser body it self requires; and therefore both

those halves, or the whole greater body, will require the moving power to be doubled.

✓ 17. THAT the moving power being in this sense doubled, should just double likewise the velocity of the same body, seems near as evident, if we consider, that the effect of the power applied must needs be the same, whether that power be applied to the body at once, or in parts. Suppose then the double power not applied to the body at once, but half of it first, and afterwards the other half; it is not conceivable for what reason the half last applied should come to have a different effect upon the body, from that which is applied first; as it must have, if the velocity of the body was not just doubled by the application of it. So far as experience can determine, we see nothing to favour such a supposition. We cannot indeed (by reason of the constant motion of the earth) make trial upon any body perfectly at rest, whereby to see whether a power applied in that case would have a different effect, from what it has, when the body is already moving; but we find no alteration in the effect of the same power on account of any difference there may be in the motion of the body, when the power is applied. The earth does not always carry bodies with the same degree of velocity; yet we find the ~~the~~ visible effects of any power applied to the same body to be at all times the very same: and a bale of goods, or other moveable body lying in a ship is as easily removed from place to place, while the ship is under sail, if its motion be steady, as when it is fixed at anchor.

18. Now this experience is alone sufficient to shew to us the whole of this law of motion.

19. SINCE we find, that the same power will always produce the same change in the motion of any body, whether that body were before moving with a swifter or slower motion; the change wrought in the motion of a body depends only on the power applied to it, without any regard to the body's former motion: and therefore the degree of motion, which the body already possesses, having no influence on the power applied to disturb its operation, the effects of the same power will not only be the same in all degrees of motion of the body; but we have likewise no reason to doubt, but that a body perfectly at rest would receive from any power as much motion, as would be equivalent to the effect of the same power applied to that body already in motion.

20. AGAIN, suppose a body being at rest, any number of equal powers should be successively applied to it; pushing it forward from time to time in the same course or direction. Upon the application of the first power the body would begin to move; when the second power was applied, it appears from what has been said, that the motion of the ~~body would~~ become double; the third power would treble the motion of the body; and so on, till after the operation of the last power the motion of the body would be as many times the motion, which the first power gave it, as there are powers in number. And the effect of this number of powers will be always the same,

same, without any regard to the space of time taken up in applying them : so that greater or lesser intervals between the application of each of these powers will produce no difference at all in their effects. Since therefore the distance of time between the action of each power is of no consequence ; without doubt the effect will still be the same, though the powers should all be applied at the very same instant ; or although a single power should be applied equal in strength to the collective force of all these powers. Hence it plainly follows, that the degree of motion, into which any body will be put out of a state of rest by any power, will be proportional to that power. A double power will give twice the velocity, a treble power three times the velocity, and so on. The foregoing reasoning will equally take place, though the body were not supposed to be at rest, when the powers began to be applied to it ; provided the direction, in which the powers were applied, either conspired with the action of the body, or was directly opposite to it. Therefore if any power be applied to a moving body, and act upon the body either in the direction wherewith the body moves, so as to accelerate the body ; or if it act directly opposite to the motion of the body, so as to retard it : in both these cases the change of motion will be proportional to the power applied ; nay, the augmentation of the motion in one case, and the diminution thereof in the other, will be equal to that degree of motion, into which the same power would put the body, had it been at rest, when the power was applied.

21. FARTHER, a power may be so applied to a moving body, as to act obliquely to the motion of the body. And the effects of such an oblique motion may be deduced from this observation; that as all bodies are continually moving along with the earth, we see that the visible effects of the same power are always the same, in whatever direction the power acts: and therefore the visible effects of any power upon a body, which seems only to be at rest, is always to appearance the same as the real effect would be upon a body truly at rest. Now suppose a body were moving along the line AB (in fig. 2.) and the eye accompanied it with an equal motion in the line CD equidistant from AB; so that when the body is at A, the eye shall be at C, and when the body is advanced to E in the line AB, the eye shall be advanced to F in the line CD, the distances AE and CF being equal. It is evident, that here the body will appear to the eye to be at rest; and the line FEG drawn from the eye through the body shall seem to the eye to be immovable; though as the body and eye move forward together, this line shall really also move; so that when the body shall be advanced to H and the eye to K, the line FEG shall be transferred into the situation KHL, this line KHL being equidistant from FEG. Now if the body when at E were to receive an impulse in the direction of the line FEG; while the eye is moving on from F to I, and carrying along with it the line FEG, the body will appear to the eye to move along this line FEG: for this is what has just now been said; that while bodies are moving along with the earth, and the spectator's eye partakes of the same motion, the effect of any power upon the body will appear to be what it

it would really have been, had the body been truly at rest when the power was applied. From hence it follows, that when the eye is advanced to K, the body will appear somewhere in the line KHL. Suppose it appear in M; then it is manifest, from what has been premised at the beginning of this paragraph, that the distance HM is equal to what the body would have run upon the line EG, during the time, wherein the eye has passed from F to K, provided that the body had been at rest, when acted upon in E. If it be farther asked, after what manner the body has moved from E to M? I answer, through a straight line; for it has been shewn above in the explication of the first law of motion, that a moving body, from the time it is left to it self, will proceed on in one continued straight line.

22. IF EN be taken equal to HM and NM be drawn; since HM is equidistant from EN, NM will be equidistant from EH. Therefore the effect of any power upon a moving body, when that power acts obliquely to the motion of the body, is to be determined in this manner. Suppose the body is moving along the straight line AEB, if when the body is come to E, a power gives it an impulse in the direction of the line EG, to find what course the body will afterwards take we must proceed thus. Take in EB any length EH, and in EG take such a length EN, that if the body had been at rest in E, the power applied to it would have caused it to move over EN in the same space of time, as it would have employed in passing over EH, if the power had not acted at all upon it. Then draw HL equidistant from EG, and NM equidistant

G
from

from E B. After this, if a line be drawn from E to the point M, where these two lines meet, the line EM will be the course into which the body will be put by the action of the power upon it at E.

23. A MATHEMATICAL reader would here expect in some particulars more regular demonstrations; but as I do not at present address my self to such, so I hope, what I have now written will render my meaning evident enough to those, who are unacquainted with that kind of reasoning.

24. Now as we have been shewing, that some actual force is necessary either to put bodies out of a state of rest into motion, or to change the motion, which they have once received; it is proper here to observe, that this quality in bodies, whereby they preserve their present state, with regard to motion or rest, till some active force disturb them, is called the *VIS INERTIAE* of matter: and by this property, matter, sluggish and unactive of it self, retains all the power impressed upon it, and cannot be made to cease from action, but by the opposition of as great a power, as that which first moved it. By the degree of this *VIS INERTIAE*, or power of inactivity, as we shall henceforth call it, we primarily judge of the quantity of solid matter in each body; for as this quality is inherent in all the bodies, upon which we can make any trial, we conclude it to be a property essential to all matter; and as we yet know no reason to suppose, that bodies are composed of different kinds of matter, we rather presume, that the matter of all bodies is the same; and that the degree of
this

this power of inactivity is in every body proportional to the quantity of the solid matter in it. But although we have no absolute proof, that all the matter in the universe is uniform, and possesses this power of inactivity in the same degree; yet we can with certainty compare together the different degrees of this power of inactivity in different bodies. Particularly this power is proportional to the weight of bodies, as Sir ISAAC NEWTON has demonstrated ^a. However, notwithstanding that this power of inactivity in any body can be more certainly known, than the quantity of solid matter in it; yet since there is no reason to suspect that one is not proportional to the other, we shall hereafter speak without hesitation of the quantity of matter in bodies, as the measure of the degree of their power of inactivity.

25. THIS being established, we may now compare the effects of the same power upon different bodies, as hitherto we have shewn the effects of different powers upon the same body. And here if we limit the word motion to the peculiar sense given to it in philosophy, we may comprehend all that is to be said upon this head under one short precept; that the same power, to whatever body it is applied, will always produce the same degree of motion. But here motion does not signify the degree of celerity or velocity with which a body moves, in which sense only we have hitherto used it; but it is made use of particularly in philosophy to signify the force with which a body moves: as if two bodies A and B be-

^a Princ. Philos. L. II. prop. 24. corol. 7. See also B. II. Ch. 5. § 3. of this treatise.

ing in motion, twice the force would be required to stop A as to stop B, the motion of A would be esteemed double the motion of B. In moving bodies, these two things are carefully to be distinguished; their velocity or celerity, which is measured by the space they pass through during any determinate portion of time; and the quantity of their motion, or the force, with which they will press against any resistance. Which force, when different bodies move with the same velocity, is proportional to the quantity of solid matter in the bodies; but if the bodies are equal, this force is proportional to their respective velocities, and in other cases it is proportional both to the quantity of solid matter in the body, and also to its velocity. To instance in two bodies A and B: if A be twice as great as B, and they have both the same velocity, the motion of A shall be double the motion of B; and if the bodies be equal, and the velocity of A be twice that of B, the motion of A shall likewise be double that of B; but if A be twice as large as B, and move twice as swift, the motion of A will be four times the motion of B; and lastly, if A be twice as large as B, and move but half as fast, the degree of their motion shall be the same.

26. THIS is the particular sense given to the word motion by philosophers, and in this sense of the word the same power always produces the same quantity or degree of motion. If the same power act upon two bodies A and B, the velocities, it shall give to each of them, shall be so adjusted to the respective bodies, that the same degree of motion shall be produced in each. If A be twice as great as B, its velocity shall be half
that

that of B; if A has three times as much solid matter as B, the velocity of A shall be one third of the velocity of B; and generally the velocity given to A shall bear the same proportion to the velocity given to B, as the quantity of solid matter contained in the body B bears to the quantity of solid matter contained in A.

27. THE reason of all this is evident from what has gone before. If a power were applied to B, which should bear the same proportion to the power applied to A, as the body B bears to A, the bodies B and A would both receive the same velocity; and the velocity, which B will receive from this power, will bear the same proportion to the velocity, which it would receive from the action of the power applied to A, as the former of these powers bears to the latter: that is, the velocity, which A receives from the power applied to it, will bear to the velocity, which B would receive from the same power, the same proportion as the body B bears to A.

28. FROM hence we may now pass to the third law of motion, where this distinction between the velocity of a body and its whole motion is farther necessary to be regarded, as shall immediately be shewn; after having first illustrated the meaning of this law by a familiar instance. If a stone or other load be drawn by a horse; the load re-acts upon the horse, as much as the horse acts upon the load; for the harness, which is strained between them, presses against the horse as much as against the load; and the progressive motion of the horse

horse forward is hindred as much by the load, as the MA as of the load is promoted by the endeavour of the horse: That is, if the horse put forth the same strength, when loosened from the load, he would move himself forwards with greater swiftness in proportion to the difference between the weight of his own body and the weight of himself and load together.

29. THIS instance will afford some general notion of the meaning of this law. But to proceed to a more philosophical explication: if a body in motion strike against another at rest, let the body striking be ever so small, yet shall it communicate some degree of motion to the body it strikes against, though the less that body be in comparison of that it impinges upon, and the less the velocity is, with which it moves, the smaller will be the motion communicated. But whatever degree of motion it gives to the resting body, the same it shall lose it self. This is the necessary consequence of the forementioned power of inactivity in matter. For suppose the two bodies equal, it is evident from the time they meet, both the bodies are to be moved by the single motion of the first; therefore the body in motion by means of its power of inactivity retaining the motion first given it, strikes upon the other with the same force, wherewith it was acted upon it self: but now both the bodies being to be moved by that force, which before moved one only, the ensuing velocity will be the same, as if the power, which was applied to one of the bodies, and put it into motion, had been applied to both; whence it appears, that they will proceed forwards, with

that half the velocity, which the body first in motion had: ~~that~~ is, the body first moved will have lost half its motion, and the other will have gained exactly as much. This rule is just, provided the bodies keep contiguous after meeting; as they would always do, if it were not for a certain cause that ~~often~~ intervenes, and which must now be explained. Bodies upon striking against each other, suffer an alteration in their figure, having their parts pressed inwards by the stroke, which for the most part recoil again afterwards, the bodies endeavouring to recover their former shape. This power, whereby bodies are enabled to regain their first figure, is usually called their elasticity, and when it acts, it forces the bodies from each other, and causes them to separate. Now the effect of this elasticity in the present case is such, that if the bodies are perfectly elastic, so as to recoil with as great a force as they are bent with, that they recover their figure in the same space of time, as has been taken up in the alteration made in it by their compression together; then this power will separate the bodies as swiftly, as they before approached, and acting upon both equally, upon the body first in motion contrary to the direction in which it moves, and upon the other as much in the direction of its motion, it will take from the first, and add to the other equal degrees of velocity: so that the power being strong enough to separate them with as great a velocity, as they approached with, the first will be quite stoppt, and that which was at rest, will receive all the motion of the other. If the bodies are elastic in a less degree, the first will not lose all its motion, nor will the other acquire the motion of the first, but fall as much short of it, as the other retains.

For

For this rule is never deviated from, that though the $\frac{1}{2}$ as of elasticity determines how much more than half its velocity the body first in motion shall lose; yet in every case the loss in the motion of this body shall be transferred to the other, that other body always receiving by the stroke as much motion, as is taken from the first.

30. THIS is the case of a body striking directly against an equal body at rest, and the reasoning here used is fully confirmed by experience. There are many other cases of bodies impinging against one another: but the mention of these shall be reserved to the next chapter, where we intend to be more particular and diffusive in the proof of these laws of motion, than we have been here.

CHAP. II.

Farther proofs of the LAWS of MOTION.

HAVING in the preceding chapter deduced the three laws of motion, delivered by our great philosopher, from the most obvious observations, that suggest them to us; I now intend to give more particular proofs of them, by recounting some of the discoveries which have been made in philosophy before Sir ISAAC NEWTON. For as they were all collected by reasoning upon those laws; so the conformity of these discoveries to experience makes them so many proofs of the truth of the principles, from which they were derived.

2. Let

that ^{of} LET us begin with the subject, which concluded the chapter. Although the body in motion be not equal to the body at rest, on which it strikes; yet the motion after the stroke is to be estimated in the same manner as above. Let A (in fig. 3.) be a body in motion towards another body B lying at rest. When A is arrived at B, it cannot proceed farther without putting B into motion; and what motion it gives to B, it must lose it self, that the whole degree of motion of A and B together, if neither of the bodies be elastic, shall be equal, after the meeting of the bodies, to the single motion of A before the stroke. Therefore, from what has been said above, it is manifest, that as soon as the two bodies are met, they will move on together with a velocity, which will bear the same proportion to the original velocity of A, as the body A bears to the sum of both the bodies.

3. IF the bodies are elastic, so that they shall separate after the stroke, A must lose a greater part of its motion, and the subsequent motion of B will be augmented by this elasticity, as much as the motion of A is diminished by it. The elasticity acting equally between both the bodies, it will communicate to each the same degree of motion; that is, it will separate the bodies by taking from the body A and adding to the body B different degrees of velocity, so proportioned to their respective quantities of matter, that the degree of motion, wherewith A separates from B, shall be equal to the degree of motion, wherewith B separates from A. It follows therefore, that the velocity taken from A by the elasticity bears to the velocity, which the same elasticity adds to B, the

same proportion, as B bears to A: consequently the velocity ^A as which the elasticity takes from A, will bear the same proportion to the whole velocity, wherewith this elasticity causes the two bodies to separate from each other, as the body B bears to the sum of the two bodies A and B; and the velocity, which is added to B by the elasticity, bears to the velocity, wherewith the bodies separate, the same proportion, as the body A bears to the sum of the two bodies A and B. Thus is found, how much the elasticity takes from the velocity of A, and adds to the velocity of B; provided the degree of elasticity be known, whereby to determine the whole velocity wherewith the bodies separate from each other after the stroke ^a.

4. AFTER this manner is determined in every case the result of a body in motion striking against another at rest. The same principles will also determine the effects, when both bodies are in motion.

5. LET two equal bodies move against each other with equal swiftness. Then the force, with which each of them presses forwards, being equal when they strike; each pressing in its own direction with the same energy, neither shall surmount the other, but both be stopt, if they be not elastic: for if they be elastic, they shall from thence recover new motion, and recede from each other, as swiftly as they met, if they be perfectly elastic; but more slowly, if less so. In the same manner, if two bodies of unequal bigness strike against each other, and their velocities be so related, that the velocity

^a How this degree of elasticity is to be found by experiment, will be shewn below in § 74.

that the lesser body shall exceed the velocity of the greater in the same proportion, as the greater body exceeds the lesser (for instance, if one body contains twice the solid matter as the other, and moves but half as fast) two such bodies will entirely suppress each other's motion, and remain from the time of their meeting fixed; if, as before, they are not elastic: but, if they are so in the highest degree, they shall recede again, each with the same velocity, wherewith they met. For this elastic power, as in the preceding case, shall renew their motion, and pressing equally upon both, shall give the same motion to both; that is, shall cause the velocity, which the lesser body receives, to bear the same proportion to the velocity, which the greater receives, as the greater body bears to the lesser: so that the velocities shall bear the same proportion to each other after the stroke, as before. Therefore if the bodies, by being perfectly elastic, have the sum of their velocities after the stroke equal to the sum of their velocities before the stroke; each body after the stroke will receive its first velocity. And the same proportion will hold likewise between the velocities, wherewith they go off, though they are elastic but in a less degree; only then the velocity of each will be less in proportion to the defect of elasticity.

6. IF the velocities, wherewith the bodies meet, are not in the proportion here supposed; but if one of the bodies, as A, has a swifter velocity in comparison to the velocity of the other; then the effect of this excess of velocity in the body A must be joined to the effect now mentioned, after the manner of this following example. Let A be twice as great as B, and

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move

move with the same swiftness as B. Here A moves with ~~the~~ as that degree of swiftness, which would answer to the foremen-
tioned proportion. For A being double to B, if it moved but with half the swiftness, wherewith B advances, it has been just now shewn, that the two bodies upon meeting would stop, if they were not elastic; and if they were elastic, ~~that~~ they would each recoil, so as to cause A to return with half the velocity, wherewith B would return. But it is evident from hence, that B by encountering A will annul half its velocity, if the bodies be not elastic; and the future motion of the bodies will be the same, as if A had advanced against B at rest with half the velocity here assigned to it. If the bodies be elastic, the velocity of A and B after the stroke may be thus discovered. As the two bodies advance against each other, the velocity, with which they meet, is made up of the velocities of both bodies added together. After the stroke their elasticity will separate them again. The degree of elasticity will determine what proportion the velocity, wherewith they separate, must bear to that, wherewith they meet. Divide this velocity, with which the bodies separate into two parts, that one of the parts bear to the other the same proportion, as the body A bears to B; and ascribe the lesser part to the greater body A, and the greater part of the velocity to the lesser body B. Then take the part ascribed to A from the common velocity, which A and B would have had after the stroke, if they had not been elastic; and add the part ascribed to B to the same common velocity. By this means the true velocities of A and B after the stroke will be made known.

that ~~IF~~ IF the bodies are perfectly elastic, the great HUYGENS ~~has~~ laid down this rule for finding their motion after concourse^a. Any straight line CD (in fig. 4, 5.) being drawn, let it be divided in E, that CE bear the same proportion to ED, as the swiftness of A bore to the swiftness of B before the stroke. Let the same line CD be also divided in F, that CF bear the same proportion to FD, as the body B bears to the body A. Then FG being taken equal to FE, if the point G falls within the line CD, both the bodies shall recoil after the stroke, and the velocity, wherewith the body A shall return, will bear the same proportion to the velocity, wherewith B shall return, as GC bears to GD; but if the point G falls without the line CD, then the bodies after their concourse shall both proceed to move the same way, and the velocity of A shall bear to the velocity of B the same proportion, that GC bears to GD, as before.

8. IF the body B had stood still, and received the impulse of the other body A upon it; the effect has been already explained in the case, when the bodies are not elastic. And ~~when~~ when they are elastic, the result of their collision is found by combining the effect of the elasticity with the other effect, in the same manner as in the last case.

9. WHEN the bodies are perfectly elastic, the rule of HUYGENS^b here is to divide the line CD (fig. 6.) in E as before, and to take EG equal to ED. And by these points

^a In oper. posthum. de Motu corpor. ex per-
fection. prop. 9.

^b In the above cited place

thus found, the motion of each body after the stroke is determined, as before.

10. IN the next place, suppose the bodies A and B were both moving the same way, but A with a swifter motion, so as to overtake B, and strike against it. The effect of the percussion or stroke, when the bodies are not elastic, is discovered by finding the common motion, which the two bodies would have after the stroke, if B were at rest, and A were to advance against it with a velocity equal to the excess of the present velocity of A above the velocity of B; and by adding to this common velocity thus found the velocity of B.

11. IF the bodies are elastic, the effect of the elasticity is to be united with this other, as in the former cases.

12. WHEN the bodies are perfectly elastic, the rule of HUYGENS^a in this case is to prolong CD (fig. 7.) and to take in it thus prolonged CE in the same proportion to ED, as the greater velocity of A bears to the lesser velocity of B; after which FG being taken equal to FE, the velocities of the two bodies after the stroke will be determined, as in the two preceding cases.

13. THUS I have given the sum of what has been written concerning the effects of percussion, when two bodies freely in motion strike directly against each other; and the results here set down, as the consequence of our reasoning

^a In the place above-cited.

from the laws of motion, answer most exactly to experience. A particular set of experiments has been invented to make trial of these effects of percussion with the greatest exactness. But I must defer these experiments, till I have explained the nature of pendulums^a. I shall therefore now proceed to describe some of the appearances, which are caused in bodies from the influence of the power of gravity united with the general laws of motion; among which the motion of the pendulum will be included,

14. THE most simple of these appearances is, when bodies fall down merely by their weight. In this case the body increases continually its velocity, during the whole time of its fall, and that in the very same proportion as the time increases. For the power of gravity acts constantly on the body with the same degree of strength: and it has been observed above in the first law of motion, that a body being once in motion will perpetually preserve that motion without the continuance of any external influence upon it: therefore, after a body has been once put in motion by the force of gravity, the body would continue that motion, though the power of gravity should cease to act any farther upon it; but, if the power of gravity continues still to draw the body down, fresh degrees of motion must continually be added to the body; and the power of gravity acting at all times with the same strength, equal degrees of motion will constantly be added in equal portions of time.

^a These experiments are described in § 73.

15. THIS conclusion is not indeed absolutely true: for we shall find hereafter^a, that the power of gravity is not of the same strength at all distances from the center of the earth. But nothing of this is the least sensible in any distance, to which we can convey bodies. The weight of bodies is the very same to sense upon the highest towers or mountains, as upon the level ground; so that in all the observations we can make, the forementioned proportion between the velocity of a falling body and the time, in which it has been descending, obtains without any the least perceptible difference.

16. FROM hence it follows, that the space, through which a body falls, is not proportional to the time of the fall; for since the body increases its velocity, a greater space will be passed over in the same portion of time at the latter part of the fall, than at the beginning. Suppose a body let fall from the point A (in fig. 8.) were to descend from A to B in any portion of time; then if in an equal portion of time it were to proceed from B to C; I say, the space BC is greater than AB; so that the time of the fall from A to C being double the time of the fall from A to B, AC shall be more than double of AB.

17. THE geometers have proved, that the spaces, through which bodies fall thus by their weight, are just in a duplicate or two-fold proportion of the times, in which the body has been falling. That is, if we were to take the line DE in the same proportion to AB, as the time, which the body has employed in falling from A to C, bears to the time of the fall

^a Book II. Chap 5.

from A to B; then AC will be to DE in the same proportion. In particular, if the time of the fall through AC be twice the time of the fall through AB; then DE will be twice AB, and AC twice DE; or AC four times AB. But if the time of the fall through AC had been thrice the time of the fall through AB; DE would have been treble of AB, and AC treble of DE; that is, AC would have been equal to nine times AB.

18. If a body fall obliquely, it will approach the ground by slower degrees, than when it falls perpendicularly. Suppose two lines AB, AC (in fig. 9.) were drawn, one perpendicular, and the other oblique to the ground DE: then if a body were to descend in the slanting line AC; because the power of gravity draws the body directly downwards, if the line AC supports the body from falling in that manner, it must take off part of the effect of the power of gravity; so that in the time, which would have been sufficient for the body to have fallen through the whole perpendicular line AB, the body shall not have passed in the line AC a length equal to AB; consequently the line AC being longer than AB, the body shall most certainly take up more time in passing through AC, than it would have done in falling perpendicularly down through AB.

19. THE geometers demonstrate, that the time, in which the body will descend through the oblique straight line AC, bears the same proportion to the time of its descent through the perpendicular AB, as the line itself AC bears to AB. And in respect to the velocity, which the body will have acquired

quired in the point C, they likewise prove, that the length of the time employed in the descent through AC so compensates the diminution of the influence of gravity from the obliquity of this line, that though the force of the power of gravity on the body is opposed by the obliquity of the line AC, yet the time of the body's descent shall be so much prolonged, that the body shall acquire the very same velocity in the point C, as it would have got at the point B by falling perpendicularly down.

20. IF a body were to descend in a crooked line, the time of its descent cannot be determined in so simple a manner; but the same property, in relation to the velocity, is demonstrated to take place in all cases: that is, in whatever line the body descends, the velocity will always be answerable to the perpendicular height, from which the body has fell. For instance, suppose the body A (in fig. 10.) were hung by a string to the pin B. If this body were let fall, till it came to the point C perpendicularly under B, it will have moved from A to C in the arch of a circle. Then the horizontal line AD being drawn, the velocity of the body in C will be the same, as if it had fallen from the point D directly down to C.

21. IF a body be thrown perpendicularly upward with any force, the velocity, wherewith the body ascends, shall continually diminish, till at length it be wholly taken away; and from that time the body will begin to fall down again, and pass over a second time in its descent the line, wherein it ascended; falling through this line with an increasing velocity in such a manner, that in every point thereof, through which

which it falls, it shall have the very same velocity, as it had in the same place, when it ascended; and consequently shall come down into the place, whence it first ascended, with the velocity which was at first given to it. Thus if a body were thrown perpendicularly up in the line AB (in fig. 11.) with such a force, as that it should stop at the point B, and there begin to fall again; when it shall have arrived in its descent to any point as C in this line, it shall there have the same velocity, as that wherewith it passed by this point C in its ascent; and at the point A it shall have gained as great a velocity, as that wherewith it was first thrown upwards. As this is demonstrated by the geometrical writers; so, I think, it will appear evident, by considering only, that while the body descends, the power of gravity must act over again, in an inverted order, all the influence it had on the body in its ascent; so as to give again to the body the same degrees of velocity, which it had taken away before.

22. AFTER the same manner, if the body were thrown upwards in the oblique straight line CA (in fig. 9.) from the point C, with such a degree of velocity as just to reach the point A; it shall by its own weight return again through the line AC by the same degrees, as it ascended.

23. AND lastly, if a body were thrown with any velocity in a line continually incurvated upwards, the like effect will be produced upon its return to the point, whence it was thrown. Suppose for instance, the body A (in fig. 12.) were hung by a string AB. Then if this body be impelled any

way, it must move in the arch of a circle. Let it receive such an impulse, as shall cause it to move in the arch AC ; and let this impulse be of such strength, that the body may be carried from A as far as D , before its motion is overcome by its weight: I say here, that the body forthwith returning from D , shall come again into the point A with the same velocity, as that wherewith it began to move.

24. It will be proper in this place to observe concerning the power of gravity, that its force upon any body does not at all depend upon the shape of the body; but that it continues constantly the same without any variation in the same body, whatever change be made in the figure of the body: and if the body be divided into any number of pieces, all those pieces shall weigh just the same, as they did, when united together in one body: and if the body be of a uniform texture, the weight of each piece will be proportional to its bulk. This has given reason to conclude, that the power of gravity acts upon bodies in proportion to the quantity of matter in them. Whence it should follow, that all bodies must fall from equal heights in the same space of time. And as we evidently see the contrary in feathers and such like substances, which fall very slowly in comparison of more solid bodies; it is reasonable to suppose, that some other cause concurs to make so manifest a difference. This cause has been found by particular experiments to be the air. The experiments for this purpose are made thus. They set up a very tall hollow glass; within which near the top they lodge a feather and some very ponderous body, usually a piece of gold,
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this metal being the most weighty of any body known to us. This glass they empty of the air contained within it, and by moving a wire, which passes through the top of the glass, they let the feather and the heavy body fall together ; and it is always found, that as the two bodies begin to descend at the same time, so they accompany each other in the fall, and come to the bottom at the very same instant, as near as the eye can judge. Thus, as far as this experiment can be depended on, it is certain, that the effect of the power of gravity upon each body is proportional to the quantity of solid matter, or to the power of inactivity in each body. For in the limited sense, which we have given above to the word motion, it has been shewn, that the same force gives to all bodies the same degree of motion, and different forces communicate different degrees of motion proportional to the respective powers^a. In this case, if the power of gravity were to act equally upon the feather, and upon the more solid body, the solid body would descend so much slower than the feather, as to have no greater degree of motion than the feather: but as both bodies descend with equal swiftness, the degree of motion in the solid body is greater than in the feather, bearing the same proportion to it, as the quantity of matter in the solid body to the quantity of matter in the feather. Therefore the effect of gravity on the solid body is greater than on the feather, in proportion to the greater degree of motion communicated ; that is, the effect of the power of gravity on the solid body bears the same proportion to its effect on the feather, as the quanti-

^a Chap. I. § 25, 26, 27, compared with § 15, &c.

ty of matter in the solid body bears to the quantity of matter in the feather. Thus it is the proper deduction from this experiment, that the power of gravity acts not on the surface of bodies only, but penetrates the bodies themselves most intimately, and operates alike on every particle of matter in them. But as the great quickness, with which the bodies fall, leaves it something uncertain, whether they do descend absolutely in the same time, or only so nearly together, that the difference in their swift motion is not discernable to the eye; this property of the power of gravity, which has here been deduced from this experiment, is farther confirmed by pendulums, whose motion is such, that a very minute difference would become sufficiently sensible. This will be farther discoursed on in another place^a; but here I shall make use of the principle now laid down to explain the nature of what is called the center of gravity in bodies.

25. THE center of gravity is that point, by which if a body be suspended, it shall hang at rest in any situation. In a globe of a uniform texture the center of gravity is the same with the center of the globe; for as the parts of the globe on every side of its center are similarly disposed, and the power of gravity acts alike on every part; it is evident, that the parts of the globe on each side of the center are drawn with equal force, and therefore neither side can yield to the other; but the globe, if supported at its center, must of necessity hang at rest. In like manner, if two equal bodies A and B (in

^a Book II. Chap. 5. § 3.

fig. 13.) be hung at the extremities of an inflexible rod CD , which should have no weight; these bodies, if the rod be supported at its middle E , shall equiponderate; and the rod remain without motion. For the bodies being equal and at the same distance from the point of support E , the power of gravity will act upon each with equal strength, and in all respects under the same circumstances; therefore the weight of one cannot overcome the weight of the other. The weight of A can no more surmount the weight of B , than the weight of B can surmount the weight of A . Again, suppose a body as AB (in fig. 14.) of a uniform texture in the form of a roller, or as it is more usually called a cylinder, lying horizontally. If a straight line be drawn between C and D , the centers of the extreme circles of this cylinder; and if this straight line, commonly called the axis of the cylinder, be divided into two equal parts in E : this point E will be the center of gravity of the cylinder. The cylinder being a uniform figure, the parts on each side the point E are equal, and situated in a perfectly similar manner; therefore this cylinder, if supported at the point E , must hang at rest, for the same reason as the inflexible rod above-mentioned will remain without motion, when suspended at its middle point. And it is evident, that the force applied to the point E , which would uphold the cylinder, must be equal to the cylinder's weight. Now suppose two cylinders of equal thickness AB and CD to be joined together at CB , so that the two axes EF , and FG lie in one straight line. Let the axis EF be divided into two equal parts at H , and the axis FG into two equal

equal parts at I. Then because the cylinder AB would be upheld at rest by a power applied in H equal to the weight of this cylinder, and the cylinder CD would likewise be upheld by a power applied in I equal to the weight of this cylinder; the whole cylinder AD will be supported by these two powers: but the whole cylinder may likewise be supported by a power applied to K, the middle point of the whole axis EG, provided that power be equal to the weight of the whole cylinder. ~~It~~ is evident therefore, that this power applied in K will produce the same effect, as the two other powers applied in H and I. It is farther to be observed, that HK is equal to half FG, and KI equal to half EF; for EK being equal to half EG, and EH equal to half EF, the remainder HK must be equal to half the remainder FG; so likewise GK being equal to half GE, and GI equal to half GF, the remainder IK must be equal to half the remainder EF. It follows therefore, that HK bears the same proportion to KI, as FG bears to EF. Besides, I believe, my readers will perceive, and it is demonstrated in form by the geometers, that the whole body of the cylinder CD bears the same proportion to the whole body of the cylinder AB, as the axis FG bears to the axis EF^a. But hence it follows, that in the two powers applied at H and I, the power applied at H bears the same proportion to the power applied at I, as KI bears to KH. Now suppose two strings HL and IM extended upwards, one from the point H and the other from I, and to be laid hold on by two powers, one strong enough to hold up the cylinder AB, and the other of

^a See Euclid's Elements, Book XII. prop. 13.

strength sufficient to support the cylinder CD. Here as these two powers uphold the whole cylinder, and therefore produce an effect, equal to what would have been produced by a power applied to the point K of sufficient force to sustain the whole cylinder: it is manifest, that if the cylinder be taken away, the axis only being left, and from the point K a string, as KN, be extended, which shall be drawn down by a power ~~equivalent~~ to the weight of the cylinder, this power shall act against the other two powers, as much as the cylinder acted against them; and consequently these three powers shall be upon a balance, and hold the axis HI fixed between them. But if these three powers preserve a mutual balance, the two powers applied to the strings HL and IM are a balance to each other; the power applied to the string HL bearing the same proportion to the power applied to the string IM, as the distance IK bears to the distance KH. Hence it farther appears, that if an inflexible rod AB (in fig. 15.) be suspended by any point C not in the middle thereof; and if at A the end of the shorter arm be hung a weight, and at B the end of the longer arm be also hung a weight less than the other, and that the greater of these weights bears to the lesser the same proportion, as the longer arm of the rod bears to the shorter; then these two weights will equiponderate: for a power applied at C equal to both these weights will support without motion the rod thus charged; since here nothing is changed from the preceding case but the situation of the powers, which are now placed on the contrary sides of the line, to which they are fixed. Also for the

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same reason, if two weights A and B (in fig. 16.) were connected together by an inflexible rod CD, drawn from C the center of gravity of A to D the center of gravity of B; and if the rod CD were to be so divided in E, that the part DE bear the same proportion to the other part CE, as the weight A bears to the weight B: then this rod being supported at E will uphold the weights, and keep them at rest without motion. This point E, by which the two bodies A and B will be supported, is called their common center of gravity. And if a greater number of bodies were joined together, the point, by which they could all be supported, is called the common center of gravity of them all. Suppose (in fig. 17.) there were three bodies A, B, C, whose respective centers of gravity were joined by the three lines DE, DF, EF: the line DE being so divided in G, that DG bear the same proportion to GE, as B bears to A; G is the center of gravity common to the two bodies A and B; that is, a power equal to the weight of both the bodies applied to G would support them, and the point G is pressed as much by the two weights A and B, as it would be, if they were both hung together at that point. Therefore, if a line be drawn from G to F, and divided in H, so that GH bear the same proportion to HF, as the weight C bears to both the weights A and B, the point H will be the common center of gravity of all the three weights; for H would be their common center of gravity, if both the weights A and B were hung together at G, and the point G is pressed as much by them in their present situation, as it would be in that case. In the same manner from the common center of these three weights;

weights, you might proceed to find the common center, if a fourth weight were added, and by a gradual progress might find the common center of gravity belonging to any number of weights whatever.

26. As all this is the obvious consequence of the proposition laid down for assigning the common center of gravity of ~~any two~~ weights, by the same proposition the center of gravity of all figures is found. In a triangle, as ABC (in fig. 18.) the center of gravity lies in the line drawn from the middle point of any one of the sides to the opposite angle, as the line BD is drawn from D the middle of the line AC to the opposite angle B ^a; so that if from the middle of either of the other sides, as from the point E in the side AB , a line be drawn, as EC , to the opposite angle; the point F , where this line crosses the other line BD , will be the center of gravity of the triangle^b. Likewise DF is equal to half FB , and EF equal to half FC ^c. In a hemisphere, as ABC (fig. 19.) if from D the center of the base the line DB be erected perpendicular to that base, and this line be so divided in E , that DE be equal to three fifths of BE , the point E is the center of gravity of the hemisphere^d.

27. IT will be of use to observe concerning the center of gravity of bodies; that since a power applied to this center alone can support a body against the power of gravity, and

^a Archimed. de æquipond. prop. 11.

^b Ibid. prop. 12.

^c Lucas Valerius De centr. gravit. fol.d. L. I.

prop. 2.

^d Idem L. II. prop. 2.

hold it fixed at rest; the effect of the power of gravity on a body is the same, as if that whole power were to exert itself on the center of gravity only. Whence it follows, that, when the power of gravity acts on a body suspended by any point, if the body is so suspended, that the center of gravity of the body can descend; the power of gravity will give motion to that body, otherwise not: or if a number of bodies are so connected together, that, when any one is put into motion, the rest shall, by the manner of their being joined, receive such motion, as shall keep their common center of gravity at rest; then the power of gravity shall not be able to produce any motion in these bodies, but in all other cases it will. Thus, if the body AB (in fig. 20, 21.) whose center of gravity is C, be hung on the point A, and the center C be perpendicularly under A (as in fig. 20.) the weight of the body will hold it still without motion, because the center C cannot descend any lower. But if the body be removed into any other situation, where the center C is not perpendicularly under A (as in fig. 21.) the body by its weight will be put into motion towards the perpendicular situation of its center of gravity. Also if two bodies A, B (in fig. 22.) be joined together by the rod CD lying in an horizontal situation, and be supported at the point E; if this point be the center of gravity common to the two bodies, their weight will not put them into motion; but if this point E is not their common center of gravity, the bodies will move; that part of the rod CD descending, in which the common center of gravity is found. So in like manner, if these two bodies were connected together by any more complex contrivance; yet, if

if one of the bodies cannot move without so moving the other, that their common center of gravity shall rest, the weight of the bodies will not put them in motion, otherwise it will.

28. I SHALL proceed in the next place to speak of the mechanical powers. These are certain instruments or machines, ~~contrived~~ for the moving great weights with small force; and their effects are all deducible from the observation we have just been making. They are usually reckoned in number five; the lever, the wheel and axis, the pulley, the wedge, and the screw; to which some add the inclined plane. As these instruments have been of very ancient use, so the celebrated ARCHIMEDES seems to have been the first, who discovered the true reason of their effects. This, I think, may be collected from what is related of him, that some expressions, which he used to denote the unlimited force of these instruments, were received as very extraordinary paradoxes: whereas to those, who had understood the cause of their great force, no expressions of that kind could have appeared surprizing.

29. ALL the effects of these powers may be judged of by this one rule, that, when two weights are applied to any of these instruments, the weights will equiponderate, if, when put into motion, their velocities will be reciprocally proportional to their respective weights. And what is said of weights, must of necessity be equally understood of any other forces
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equivalent to weights, such as the force of a man's arm, a stream of water, or the like.

30. BUT to comprehend the meaning of this rule, the reader must know, what is to be understood by reciprocal proportion; which I shall now endeavour to explain, as distinctly as I can; for I shall be obliged very frequently to make use of this term. When any two things are so related, that one increases in the same proportion as the other, they are directly proportional. So if any number of men can perform in a determined space of time a certain quantity of any work, suppose drain a fish-pond, or the like; and twice the number of men can perform twice the quantity of the same work, in the same time; and three times the number of men can perform as soon thrice the work; here the number of men and the quantity of the work are directly proportional. On the other hand, when two things are so related, that one decreases in the same proportion, as the other increases, they are said to be reciprocally proportional. Thus if twice the number of men can perform the same work in half the time, and three times the number of men can finish the same in a third part of the time; then the number of men and the time are reciprocally proportional. We shewed above ^a how to find the common center of gravity of two bodies, there the distances of that common center from the centers of gravity of the two bodies are reciprocally proportional to the respective bodies. For CE in fig. 16. being in the same pro-

^a § 25.

portion

portion to ED, as B bears to A ; CE is so much greater in proportion than ED, as A is less in proportion than B.

31. Now this being understood, the reason of the rule here stated will easily appear. For if these two bodies were put in motion, while the point E rested, the velocity, wherewith A would move, would bear the same proportion to the velocity, wherewith B would move, as EC bears to ED. The velocity therefore of each body, when the common center of gravity rests, is reciprocally proportional to the body. But we have shewn above^a, that if two bodies are so connected together, that the putting them in motion will not move their common center of gravity ; the weight of those bodies will not produce in them any motion. Therefore in any of these mechanical engines, if, when the bodies are put into motion, their velocities are reciprocally proportional to their respective weights, whereby the common center of gravity would remain at rest, the bodies will not receive any motion from their weight, that is, they will equiponderate. But this perhaps will be yet more clearly conceived by the particular description of each mechanical power.

32. THE lever was first named above. This is a bar made use of to sustain and move great weights. The bar is applied in one part to some strong support ; as the bar AB (in fig. 23, 24.) is applied at the point C to the support D. In some other part of the bar, as E, is applied the weight to be sustained or moved ; and in a third place, as F, is applied another weight or equivalent force, which is to sustain or move the

the weight at E. Now here, if, when the lever should be put in motion, and turned upon the point C, the velocity, wherewith the point F would move, bears the same proportion to the velocity, wherewith the point E would move, as the weight at E bears to the weight or force at F; then the lever thus charged will have no propensity to move either way. If the weight or other force at F be not so great as to bear this proportion, the weight at E will not be sustained; but if the force at F be greater than this, the weight at E will be furmounted. This is evident from what has been said above^a, when the forces at E and F are placed (as in fig. 23.) on different sides of the support D. It will appear also equally manifest in the other case, by continuing the bar BC in fig. 24. on the other side of the support D, till CG be equal to CF, and by hanging at G a weight equivalent to the power at F; for then, if the power at F were removed, the two weights at G and E would counterpoize each other, as in the former case: and it is evident, that the point F will be lifted up by the weight at G with the same degree of force, as by the other power applied to F; since, if the weight at E were removed, a weight hung at F equal to that at G would balance the lever, the distances CG and CF being equal.

33. IF the two weights, or other powers, applied to the lever do not counterbalance each other; a third power may be applied in any place proposed of the lever, which shall

^a Pag. 65, 68.

hold the whole in a just counterpoize. Suppose (in fig. 25.) the two powers at E and F did not equiponderate, and it were required to apply a third power to the point G, that might be sufficient to balance the lever. Find what power in F would just counterbalance the power in E; then if the difference between this power and that, which is actually applied at F, bear the same proportion to the third power to be applied at G, as the distance CG bears to CF; the lever will be counterpoized by the help of this third power, if it be so applied as to act the same way with the power in F, when that power is too small to counterbalance the power in E; but otherwise the power in G must be so applied, as to act against the power in F. In like manner, if a lever were charged with three, or any greater number of weights or other powers, which did not counterpoize each other, another power might be applied in any place proposed, which should bring the whole to a just balance. And what is here said concerning a plurality of powers, may be equally applied to all the following cases.

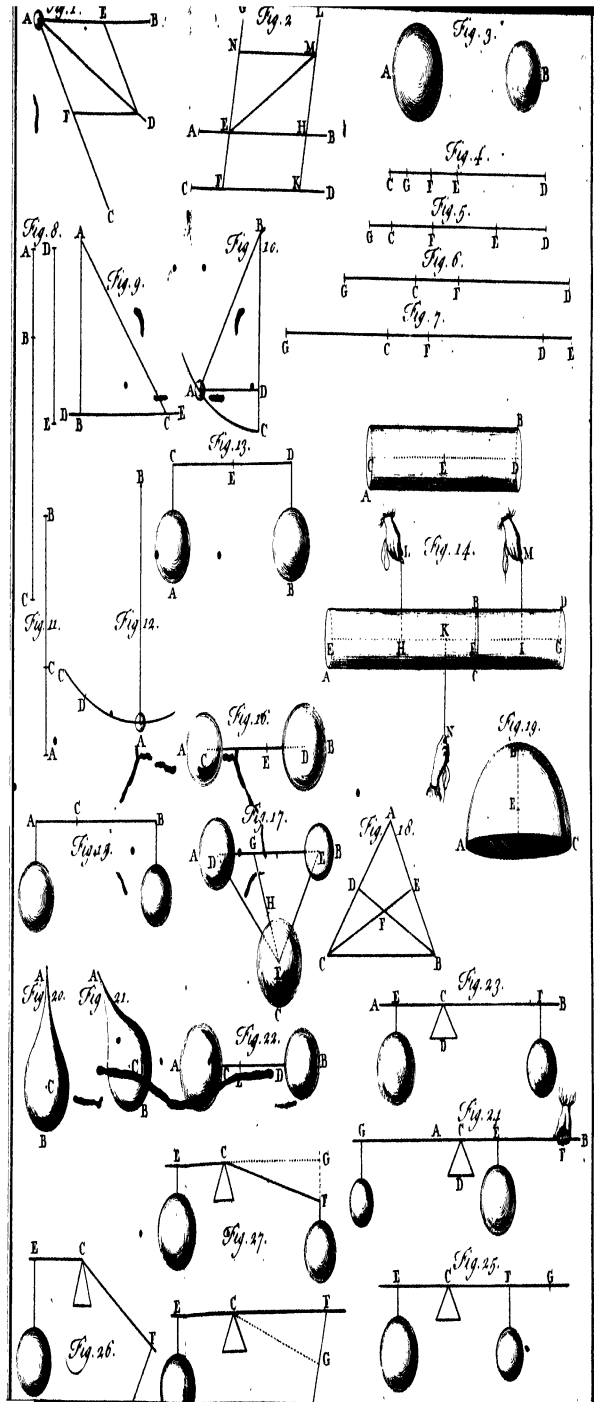
34. IF the lever should consist of two arms making an angle at the point C (as in fig. 26.) yet if the forces are applied perpendicularly to each arm, the same proportion will hold between the forces applied, and the distances of the center, whereon the lever rests, from the points to which they are applied. That is, the weight at E will be to the force in F in the same proportion, as CF bears to CE.

35. BUT whenever the forces applied to the lever act obliquely to the arm, to which they are applied (as in fig. 27.)

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then the strength of the forces is to be estimated by lines let fall from the center of the lever to the directions, wherein the forces act. To balance the levers in fig. 27, the weight or other force at F will bear the same proportion to the weight at E, as the distance CE bears to CG the perpendicular let fall from the point C upon the line, which denotes the direction wherein the force applied to F acts: for here, if the lever be put into motion, the power applied to F will begin to move in the direction of the line FG; and therefore its first motion will be the same, as the motion of the point G.

36. WHEN two weights hang upon a lever, and the point, by which the lever is supported, is placed in the middle between the two weights, that the arms of the lever are both of equal length; then this lever is particularly called a balance; and equal weights equiponderate as in common scales. When the point of support is not equally distant from both weights, it constitutes that instrument for weighing, which is called a steelyard. Though both in common scales, and the steelyard, the point, on which the beam is hung, is not usually placed just in the same straight line with the points, that hold the weights, but rather a little above (as in fig. 28.) where the lines drawn from the point C, whereon the beam is suspended, to the points E and F, on which the weights are hung, do not make absolutely one continued line. If the three points E, C, and F were in one straight line, those weights, which equiponderated, when the beam hung horizontally, would also equiponderate in any other situation. But we see in these instruments, when they are charged with weights, which



Which equiponderate with the beam hanging horizontally ; that, if the beam be inclined either way, the weight most elevated furmounts the other, and descends, causing the beam to swing, till by degrees it recovers its horizontal position. This effect arises from the forementioned structure : for by this structure these instruments are levers composed of two arms, which make an angle at the point of support (as in fig. 29, 30.) the first of which represents the case of the common balance, the second the case of the steelyard. In the first, where CE and CF are equal, equal weights hung at E and F will equiponderate, when the points E and F are in an horizontal situation. Suppose the lines EG and FH to be perpendicular to the horizon, then they will denote the directions, wherein the forces applied to E and F act. Therefore the proportion between the weights at E and F, which shall equiponderate, are to be judged of by perpendiculars, as CI, CK, let fall from C upon EG and FH : so that the weights being equal, the lines CI, CK, must be equal also, when the weights equiponderate. But I believe my readers will easily see, that since CE and CF are equal, the lines CI and CK will be equal, when the points E and F are horizontally situated.

37. IF this lever be set into any other position (as in fig. 31.) then the weight, which is raised highest, will outweigh the other. Here, if the point F be raised higher than E, the perpendicular CK will be longer than CI: and therefore the weights would equiponderate, if the weight at F

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were



were less than the weight at E. But the weight at F is equal to that at E; therefore is greater, than is necessary to counter-balance the weight at E, and consequently will outweigh it, and draw the beam of the lever down.

38. IN like manner in the case of the steelyard (fig. 32.) if the weights at E and F are so proportioned, as to equiponderate, when the points E and F are horizontally situated; then in any other situation of this lever the weight, which is raised highest, will preponderate. That is, if in the horizontal situation of the points E and F the weight at F bears the same proportion to the weight at E, as CI bears to CK; then, if the point F be raised higher than E (as in fig. 32.) the weight at F shall bear a greater proportion to the weight at E, than CI bears to CK.

39. FARTHER a lever may be hung upon an axis, and then the two arms of the lever need not be continuous, but fixed to different parts of this axis; as in fig. 33, where the axis AB is supported by its two extremities A and B. To this axis one arm of the lever is fixed at the point C, the other at the point D. Now here, if a weight be hung at E, the extremity of that arm, which is fixed to the axis at the point C; and another weight be hung at F, the extremity of the arm, which is fixed on the axis at D; then these weights will equiponderate, when the weight at E bears the same proportion to the weight at F, as the arm DF bears to CE.

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40. THIS is the case, if both the arms are perpendicular to the axis, and lie (as the geometers express themselves) in the same plane; or, in other words, if the arms are so fixed ~~ed~~ perpendicularly upon the axis, that, when one of them lies horizontally, the other shall also be horizontal. If either arm ~~is~~ ^{is} not perpendicular to the axis; then, in determining the proportion between the weights, instead of the length of that arm, you must use the perpendicular let fall upon the axis from the extremity of that arm. If the arms are not so fixed as to become horizontal, at the same time; the method of assigning the proportion between the weights is analogous to that made use of above in levers, which make an angle at the point, whereon they are supported.

41. FROM this case of the lever hung on an axis, it is easy to make a transition to another mechanical power, the wheel and axis.

42. THIS instrument is a wheel fixed on a roller, the roller being supported at each extremity so as to turn round freely with the wheel, in the manner represented in fig. 34, where AB is the wheel, CD the roller, and EF its two supports. Now suppose a weight G hung by a cord wound round the roller, and another weight H hung by a cord wound about the wheel the contrary way: that these weights ~~may~~ support each other, the weight H must bear the same proportion to the weight G, as the thickness of the roller bears to the diameter of the wheel.

43. SUPPOSE

43. SUPPOSE the line kl to be drawn through the middle of the roller; and from the place of the roller, where the cord, on which the weight G hangs, begins to leave the roller, as at m , let the line mn be drawn perpendicularly to kl ; and from the point, where the cord holding the weight H begins to leave the wheel, as at o , let the line op be drawn perpendicular to kl . This being done, the two lines op and mn represent two arms of a lever fixed on the axis kl ; consequently the weight H will bear to the weight G the same proportion, as mn bears to op . But mn bears the same proportion to op , as the thickness of the roller bears to the diameter of the wheel; for mn is half the thickness of the roller, and op half the diameter of the wheel.

44. IF the wheel be put into motion, and turned once round, that the cord, on which the weight G hangs, be wound once more round the axis; then at the same time the cord, whereon the weight H hangs, will be wound off from the wheel one circuit. Therefore the velocity of the weight G will bear the same proportion to the velocity of the weight H , as the circumference of the roller to the circumference of the wheel. But the circumference of the roller bears the same proportion to the circumference of the wheel, as the thickness of the roller bears to the diameter of the wheel, consequently the velocity of the weight G bears to the velocity of the weight H the same proportion, as the thickness of the roller bears to the diameter of the wheel, which is the proportion that the weight H bears to the weight G . Therefore as before in the lever, so here also the general rule laid down

down above is verified, that the weights equiponderate, when their velocities would be reciprocally proportional to their respective weights.

45. IN like manner, if on the same axis two wheels of different sizes are fixed (as in fig. 35.) and a weight hung on each; the weights will equiponderate, if the weight hung on the greater wheel bear the same proportion to the weight hung on the lesser, as the diameter of the lesser wheel bears to the diameter of the greater.

46. IT is usual to join many wheels together in the same frame, which by the means of certain teeth, formed in the circumference of each wheel, shall communicate motion to each other. A machine of this nature is represented in fig. 36. Here ABC is a winch, upon which is fixed a small wheel D indented with teeth, which move in the like teeth of a larger wheel EF fixed on the axis GH. Let this axis carry another wheel I, which shall move in like manner a greater wheel KL fixed on the axis MN. Let this axis carry another small wheel O, which after the same manner shall turn about a larger wheel PQ fixed on the roller RS, on which a cord shall be wound, that holds a weight, as T. Now the proportion required between the weight T and a power applied to the winch at A sufficient to support the weight, will most easily be estimated, by computing the proportion, which the velocity of the point A would bear to the velocity of the weight. If the winch be turned round, the point A will describe a circle as AV. Suppose the wheel EF to have ten times the number of teeth, as the

the wheel D; then the winch must turn round ten times to carry the wheel EF once round. If the wheel KL has also ten times the number of teeth, as I, the wheel I must turn round ten times to carry the wheel KL once round; and consequently the winch ABC must turn round an hundred times to turn the wheel KL once round. Lastly, if the wheel PQ has ten times the number of teeth, as the wheel O, the winch must turn about one thousand times in order to turn the wheel PQ, or the roller RS once round. Therefore here the point A must have gone over the circle AV a thousand times, in order to lift the weight T through a space equal to the circumference of the roller RS: whence it follows, that the power applied at A will balance the weight T, if it bear the same proportion to it, as the circumference of the roller to one thousand times the circle AV; or the same proportion as half the thickness of the roller bears to one thousand times AB.

47. I SHALL now explain the effect of the pulley. Let a weight hang by a pulley, as in fig. 37. Here it is evident, that the power A, by which the weight B is supported, must be equal to the weight; for the cord CD is equally strained between them; and if the weight B move, the power A must move with equal velocity. The pulley E has no other effect, than to permit the power A to act in another direction, than it must have done, if it had been directly applied to support the weight without the intervention of any such instrument.

48. AGAIN, let a weight be supported, as in fig. 38; where the weight A is fixed to the pulley B, and the cord, which

which the weight is upheld, is annexed by one extremity to a hook C, and at the other end is held by the power D. Here the weight is supported by a cord doubled; infomuch that although the cord were not strong enough to hold the weight single, yet being thus doubled it might support it. If the end of the cord held by the power D were hung on the hook C, as well as the other end; then, when both ends of the cord were tied to the hook, it is evident, that the hook would bear the whole weight; and each end of the string would bear against the hook with the force of half the weight only, seeing both ends together bear with the force of the whole. Hence it is evident, that, when the power D holds one end of the weight, the force, which it must exert to support the weight, must be equal to just half the weight. And the same proportion between the weight and power might be collected from comparing the respective velocities, with which they would move; for it is evident, that the power must move through a space equal to twice the distance of the pulley from the hook, in order to lift the pulley up to the hook.

49. IT is equally easy to estimate the effect, when many pulleys are combined together, as in fig. 39, 40; in the first of which the under set of pulleys, and consequently the weight is held by six strings; and in the latter figure by five: therefore in the first of these figures the power to support the weight, must be one sixth part only of the weight, and in the latter figure the power must be one fifth part.

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SO. THERE

50. THERE are two other ways of supporting a weight by pulleys, which I shall particularly consider.

51. ONE of these ways is represented in fig. 41. Here the weight being connected to the pulley B, a power equal to half the weight A would support the pulley C, if applied immediately to it. Therefore the pulley C is drawn down with a force equal to half the weight A. But if the pulley D were to be immediately supported by half the force, with which the pulley C is drawn down, this pulley D will uphold the pulley C; so that if the pulley D be upheld with a force equal to one fourth part of the weight A, that force will support the weight. But, for the same reason as before, if the power in E be equal to half the force necessary to uphold the pulley D; this pulley, and consequently the weight A, will be upheld: therefore, if the power in E be one eighth part of the weight A, it will support the weight.

52. ANOTHER way of applying pulleys to a weight is represented in fig. 42. To explain the effect of pulleys thus applied, it will be proper to consider different weights hanging, as in fig. 43. Here, if the power and weights balance each other, the power A is equal to the weight B; the weight C is equal to twice the power A, or the weight B; and for the same reason the weight D is equal to twice the weight C, or equal to four times the power A. It is evident therefore, that all the three weights B, C, D together are equal to seven times the power A. But if these three weights were joined in one, they would produce the case of fig. 40: so that in that figure the weight

weight A, where there are three pulleys, is seven times the power B. If there had been but two pulleys, the weight would have been three times the power ; and if there had ben four pulleys, the weight would have been fifteen times the power.

§3. THE wedge is next to be considered. The form of this instrument is sufficiently known. When it is put under any weight (as in fig. 44.) the force, with which the wedge will lift the weight, when drove under it by a blow upon the end A B, will bear the same proportion to the force, wherewith the blow would act on the weight, if directly applied to it ; as the velocity, which the wedge receives from the blow, bears to the velocity, wherewith the weight is lifted by the wedge.

§4. THE screw is the fifth mechanical power. There are two ways of applying this instrument. Sometimes it is screwed into a hole, as in fig. 45, where the screw A B is screwed through the plank C D. Sometimes the screw is applied to the teeth of a wheel, as in fig. 46, where the thred of the screw A B turns in the teeth of a wheel C D. In both these cases, if a bar, as A E, be fixed to the end A of the screw ; the force, wherewith the end B of the screw in fig. 45 is forced down, and the force, wherewith the teeth of the wheel C D in fig. 44 are held, bears the same proportion to the power applied to the end E of the bar ; as the velocity, wherewith the end E will move, when the screw is turned, bears to the velocity, wherewith the end B of the screw in fig. 43, or the teeth of the wheel C D in fig. 46, will be moved.

55. THE inclined plane affords also a means of raising a weight with less force, than what is equal to the weight itself. Suppose it were required to raise the globe A (in fig. 47.) from the ground BC up to the point, whose perpendicular height from the ground is ED. If this globe be drawn along the slant DF, less force will be required to raise it, than if it were lifted directly up. Here if the force applied to the globe bear the same proportion only to its weight, as ED bears to FD, it will be sufficient to hold up the globe; and therefore any addition to that force will put it in motion, and draw it up; unless the globe, by pressing against the plane, whereon it lies, adhere in some degree to the plane. This indeed it must always do more or less, since no plane can be made so absolutely smooth as to have no inequalities at all; nor yet so infinitely hard, as not to yield in the least to the pressure of the weight. Therefore the globe cannot be laid on such a plane, whereon it will slide with perfect freedom, but they must in some measure rub against each other; and this friction will make it necessary to employ a certain degree of force more, than what is necessary to support the globe, in order to give it any motion. But as all the mechanical powers are subject in some degree or other to the like impediment from friction; I shall here only shew what force would be necessary to sustain the globe, if it could lie upon the plane without causing any friction at all. And I say, that if the globe were drawn by the cord GH, lying parallel to the plane DF; and the force, wherewith the cord is pulled, bear the same proportion to the weight of the globe, as ED bears to DF;
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This force will sustain the globe. In order to the making proof of this, let the cord GH be continued on, and turned over the pulley I, and let the weight K be hung to it. Now I say, if this weight bears the same proportion to the globe A, as DE bears to DF, the weight will support the globe. I think it is very manifest, that the center of the globe A will lie in one continued line with the cord HG. Let L be the center of the globe, and M the center of gravity of the weight K. In the first place let the weight hang so, that a line drawn from L to M shall lie horizontally; and I say, if the globe be moved either up or down the plane DF, the weight will so move along with it, that the center of gravity common to both the weights shall continue in this line LM, and therefore shall in no case descend. To prove this more fully, I shall depart a little from the method of this treatise, and make use of a mathematical proposition or two: but they are such, as any person, who has read EUCLID'S ELEMENTS, will fully comprehend; and are in themselves so evident, that, I believe, my readers, who are wholly strangers to geometrical writings, will make no difficulty of admitting them. This being premised, let the globe be moved up, till its center be at G, then will M the center of gravity of the weight K be sunk to N; so that MN shall be equal to GL. Draw NG crossing the line ML in O; then I say, that O is the common center of gravity of the two weights in this their new situation. Let GP be drawn perpendicular to ML; then GL will bear the same proportion to GP, as DF bears to DE; and MN being equal to GL, MN will bear the same proportion
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to GP , as DF bears to DE . But NO bears the same proportion to OG , as MN bears to GP ; consequently NO will bear the same proportion to OG , as DF bears to DE . In the last place, the weight of the globe A bears the same proportion to the other weight K , as DF bears to DE ; therefore NO bears the same proportion to OG , as the weight of the globe A bears to the weight K . Whence it follows, that, when the center of the globe A is in G , and the center of gravity of the weight K is in N , O will be the center of gravity common to both the weights. After the same manner, if the globe had been caused to descend, the common center of gravity would have been found in this line ML . Since therefore no motion of the globe either way will make the common center of gravity descend, it is manifest, from what has been said above, that the weights A and K counterpoize each other.

§6. I SHALL now consider the case of pendulums. A pendulum is made by hanging a weight to a line, so that it may swing backwards and forwards. This motion the geometers have very carefully considered, because it is the most commodious instrument of any for the exact measurement of time.

§7. I HAVE observed already^a, that if a body hanging perpendicularly by a string, as the body A , (in fig. 48.) hangs by the string AB , be put so into motion, as to be made to ascend up the circular arch AC ; then as soon as it has arrived

at the highest point, to which the motion, that the body has received, will carry it; it will immediately begin to descend, and at A will receive again as great a degree of motion, as it had at first. This motion therefore will carry the body up the arch AD, as high as it ascended before in the arch AC. Consequently in its return through the arch DA it will acquire again at A its original velocity, and advance a second time up the arch AC as high as at first; by this means continuing without end its reciprocal motion. It is true indeed, that in fact every pendulum, which we can put in motion, will gradually lessen its swing, and at length stop, unless there be some power constantly applied to it, whereby its motion shall be renewed; but this arises from the resistance, which the body meets with both from the air, and the string by which it is hung: for as the air will give some obstruction to the progress of the body moving through it; so also the string, whereon the body hangs, will be a farther impediment; for this string must either slide on the pin, whereon it hangs, or it must bend to the motion of the weight; in the first there must be some degree of friction; and in the latter the string will make some resistance to its inflection. However, if all resistance could be removed, the motion of a pendulum would be perpetual.

§8. BUT to proceed, the first property, I shall take notice of in this motion, is, that the greater arch the pendulous body moves through, the greater time it takes up: though the length of time does not increase in so great a proportion as the arch. Thus if CD be a greater arch, and EF a lesser, where CA is equal to AD, and EA equal to AF; the body, when

when it swings through the greater arch CD , shall take up in its swing from C to D a longer time than in swinging from E to F , when it moves only in that lesser arch; or the time in which the body let fall from C will descend through the arch CA is greater than the time, in which it will descend through the arch EA , when let fall from E . But the first of these times will not hold the same proportion to the latter, as the first arch CA bears to the other arch EA ; which will appear thus. Let CG and EH be two horizontal lines. It has been remarked above^a, that the body in falling through the arch CA will acquire as great a velocity at the point A , as it would have gained by falling directly down through GA ; and in falling through the arch EA it will acquire in the point A only that velocity, which it would have got in falling through HA . Therefore, when the body descends through the greater arch CA , it shall gain a greater velocity, than when it passes only through the lesser; so that this greater velocity will in some degree compensate the greater length of the arch.

59. THE increase of velocity, which the body acquires in falling from a greater height, has such an effect, that, if straight lines be drawn from A to C and E , the body would fall through the longer straight line CA just in the same time, as through the shorter straight line EA . This is demonstrated by the geometers, who prove, that if any circle, as $ABCD$ (fig. 49.) be placed in a perpendicular situation; a body shall fall obliquely through every line, as AB drawn from the lowest point A in the circle to any other point in the circum-

^a § 20.

ference just in the same time, as would be employed by the body in falling perpendicularly down through the diameter CA. But the time in which the body will descend through the arch, is different from the time, which it would take up in falling through the line AB.

60. IT has been thought by some, that because in very small arches, this correspondent straight line differs but little from the arch itself; therefore the descent through this straight line would be performed in such small arches nearly in the same time as through the arches themselves: so that if a pendulum were to swing in small arches, half the time of a single swing would be nearly equal to the time, in which a body would fall perpendicularly through twice the length of the pendulum. That is, the whole time of the swing, according to this opinion, will be four fold the time required for the body to fall through half the length of the pendulum; because the time of the body's falling down twice the length of the pendulum is half the time required for the fall through one quarter of this space, that is through half the pendulum's length. However there is here a mistake; for the whole time of the swing, when the pendulum moves through small arches, bears to the time required for a body to fall down through half the length of the pendulum very nearly the same proportion, as the circumference of a circle bears to its diameter; that is very nearly the proportion of 355 to 113, or little more than the proportion of 3 to 1. If the pendulum takes so great a swing, as to pass over an arch equal to one sixth part of the whole circumference of the

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circle,

circle, it will swing 115 times, while it ought according to this proportion to have swung 117 times; so that, when it swings in so large an arch, it loses something less than two swings in an hundred. If it swing through $\frac{1}{10}$ only of the circle, it shall not lose above one vibration in 160. If it swing in $\frac{1}{20}$ of the circle, it shall lose about one vibration in 690. If its swing be confined to $\frac{1}{40}$ of the whole circle, it shall lose very little more than one swing in 2600. And if it take no greater a swing than through $\frac{1}{80}$ of the whole circle, it shall not lose one swing in 5800.

61. Now it follows from hence, that, when pendulums swing in small arches, there is very nearly a constant proportion observed between the time of their swing, and the time, in which a body would fall perpendicularly down through half their length. And we have declared above, that the spaces, through which bodies fall, are in a two fold proportion of the times, which they take up in falling^a. Therefore in pendulums of different lengths, swinging through small arches, the lengths of the pendulums are in a two fold or duplicate proportion of the times, they take in swinging; so that a pendulum of four times the length of another shall take up twice the time in each swing, one of nine times the length will make one swing only for three swings of the shorter, and so on.

62. THIS proportion in the swings of different pendulums not only holds in small arches; but in large ones also,

provided they be such, as the geometers call similar ; that is, if the arches bear the same proportion to the whole circumferences of their respective circles. Suppose (in fig. 48.) AB, CD to be two pendulums. Let the arch EF be described by the motion of the pendulum AB , and the arch GH be described by the pendulum CD ; and let the arch EF bear the same proportion to the whole circumference, which would be formed by turning the pendulum AB quite round about the point A , as the arch GH bears to the whole circumference, that would be formed by turning the pendulum CD quite round the point C . Then I say, the proportion, which the length of the pendulum AB bears to the length of the pendulum CD , will be two fold of the proportion, which the time taken up in the description of the arch EF bears to the time employed in the description of the arch GH .

63. THUS pendulums, which swing in very small arches, are nearly an equal measure of time. But as they are not such an equal measure to geometrical exactness ; the mathematicians have found out a method of causing a pendulum so to swing, that, if its motion were not obstructed by any resistance, it would always perform each swing in the same time, whether it moved through a greater, or a lesser space. This was first discovered by the great HUYGENS, and is as follows. Upon the straight line AB (in fig. 49.) let the circle CDE be so placed, as to touch the straight line in the point C . Then let this circle roll along upon the straight line AB , as a coach-wheel rolls along upon the ground. It is evident, that, as

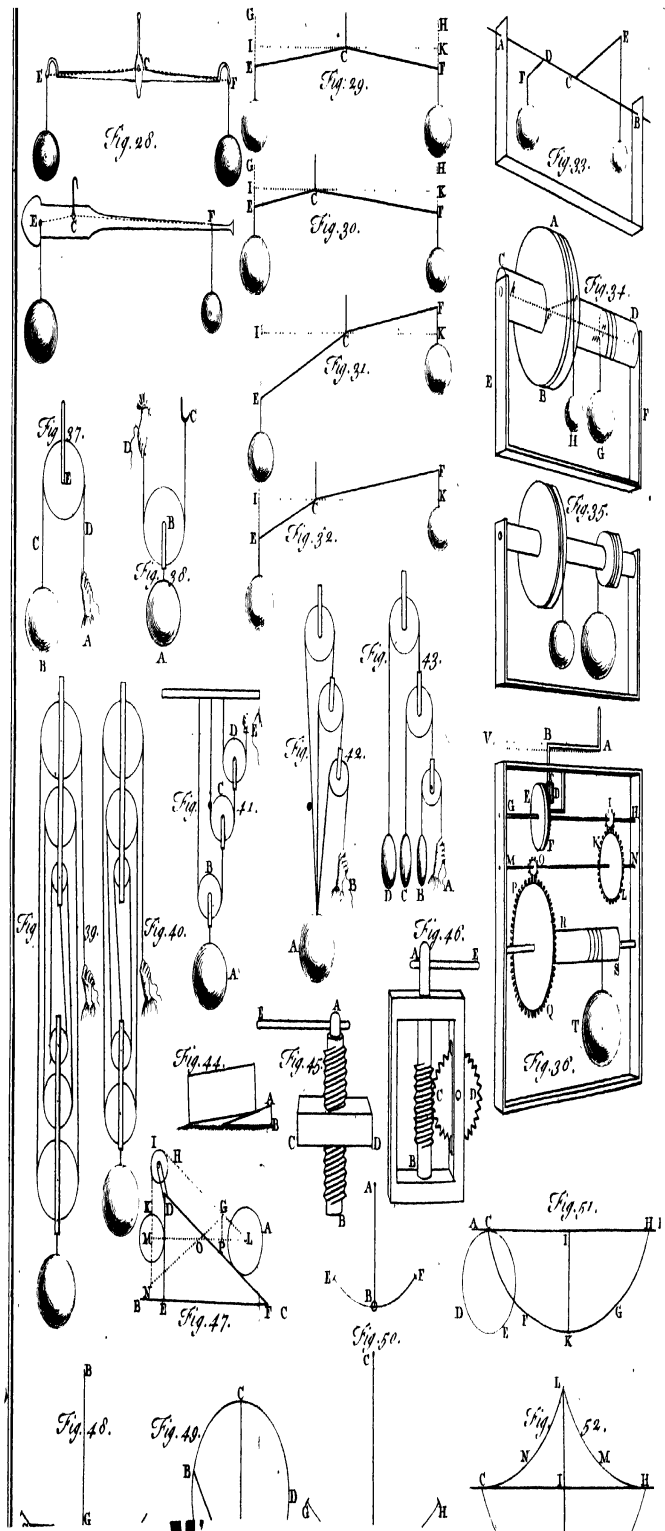
soon as ever the circle begins to move, the point C in the circle will be lifted off from the straight line AB; and in the motion of the circle will describe a crooked course, which is represented by the line CFGH. Here the part CH of the straight line included between the two extremities C and H of the line CFGH will be equal to the whole circumference of the circle CDE; and if CH be divided into two equal parts at the point I, and the straight line IK be drawn perpendicular to CH, this line IK will be equal to the diameter of the circle CDE. Now in this line if a body were to be let fall from the point H, and were to be carried by its weight down the line HGK, as far as the point K, which is the lowest point of the line CFGH; and if from any other point G a body were to be let fall in the same manner; this body, which falls from G, will take just the same time in coming to K, as the body takes up, which falls from H. Therefore if a pendulum can be so hung, that the ball shall move in the line AGFE, all its swings, whether long or short, will be performed in the same time; for the time, in which the ball will descend to the point K, is always half the time of the whole swing. But the ball of a pendulum will be made to swing in this line by the following means. Let KI (in fig. 52.) be prolonged upwards to L, till IL is equal to IK. Then let the line LMH equal and like to KH be applied, as in the figure between the points L and H, so that the point which in this line LMH answers to the point H in the line KH shall be applied to the point L, and the point answering to the point K shall be applied to the point H. Also let such another line LNC be applied between L and C in the same manner.

manner. This preparation being made ; if a pendulum be hung at the point L of such a length, that the ball thereof shall reach to K ; and if the string shall continually bend against the lines HML and LNC, as the pendulum swings to and fro ; by this means the ball shall constantly keep in the line CKH.

64. Now in this pendulum, as all the swings, whether long or short, will be performed in the same time ; so the time of each will exactly bear the same proportion to the time required for a body to fall perpendicularly down, through half the length of the pendulum, that is from I to K, as the circumference of a circle bears to its diameter.

65. It may from hence be understood in some measure, why, when pendulums swing in circular arches, the times of their swings are nearly equal, if the arches are small, though those arches be of very unequal lengths ; for if with the semidiameter LK the circular arch OKP be described, this arch in the lower part of it will differ very little from the line CKH.

66. It may not be amiss here to remark, that a body will fall in this line CKH (fig. 53.) from C to any other point, as Q or R in a shorter space of time, than if it moved through the straight line drawn from C to the other point ; or through any other line whatever, that can be drawn between these two points.



been observed above, that the effect of gravity upon any body is the same, as if the whole force were exerted on the body's center of gravity ^a. Since therefore the power of gravity in drawing down the body must also communicate to it the rolling motion just described ; it seems evident, that the center of gravity of the body cannot be drawn down as swiftly, as when the power of gravity has no other effect to produce on the body, than merely to draw it downward. If therefore the whole matter of the body CB could be crowded into its center of gravity, so that being united into one point, this rolling motion here mentioned might give no hindrance to its descent ; this center would descend faster, than it can now do. And the point, which now descends as fast, as if the whole matter of the body CB were crowded into it, will be farther removed from the point A, than the center of gravity of the body CB.

69. AGAIN, suppose the pendulum AB (in fig. 55.) to hang obliquely. Here the power of gravity will operate less upon the ball of the pendulum, than before : but the line DE being drawn so, as to stand perpendicular to the rod AC of the pendulum ; the force of gravity upon the body CB, now it is in this situation, will produce the same effect, as if the body were to glide down an inclined plane in the position of DE. But here the motion of the body, when the rod is fixed to the point A, will not be equal to the uninterrupted descent of the body down this plane ; for the body

will here also receive the same kind of rotation in its motion, as before ; so that the motion of the center of gravity will in like manner be retarded ; and the point, which here descends with that degree of swiftness, which the body would have, if not hindered by being fixed to the point A ; that is, the point, which descends as fast, as if the whole body were crowded into it, will be as far removed from the point A, as before.

70. THIS point, by which the length of the pendulum is to be estimated, is called the center of oscillation. . And the mathematicians have laid down general directions, whereby to find this center in all bodies. If the globe AB (in fig. 56.) be hung by the string CD, whose weight need not be regarded, the center of oscillation is found thus. Let the straight line drawn from C to D be continued through the globe to F. That it will pass through the center of the globe is evident. Suppose E to be this center of the globe ; and take the line G of such a length, that it shall bear the same proportion to ED, as ED bears to EC. Then EH being made equal to $\frac{2}{3}$ of G, the point H shall be the center of oscillation^a. If the weight of the rod CD is too considerable to be neglected, divide CD (fig. 57) in I, that DI be equal to $\frac{1}{3}$ part of CD ; and take K in the same proportion to CI, as the weight of the globe AB to the weight of the rod CD. Then having found H, the center of oscillation of the globe, as before, divide IK in L, so that IL shall bear the same pro-

^a Hugen. Horolog. oscillat. pag. 141, 142.

portion

portion to LH , as the line CH bears to K ; and L shall be the center of oscillation of the whole pendulum.

71. THIS computation is made upon supposition, that the center of oscillation of the rod CD , if that were to swing alone without any other weight annexed, would be the point I . And this point would be the true center of oscillation, so far as the thickness of the rod is not to be regarded. If any one chuses to take into consideration the thickness of the rod, he must place the center of oscillation thereof so much below the point I , that eight times the distance of the center from the point I shall bear the same proportion to the thickness of the rod, as the thickness of the rod bears to its length CD ^a.

72. IT has been observed above, that when a pendulum swings in an arch of a circle, as here in fig. 58, the pendulum AB swings in the circular arch CD ; if you draw an horizontal line, as EF , from the place whence the pendulum is let fall, to the line AG , which is perpendicular to the horizon: then the velocity, which the pendulum will acquire in coming to the point G , will be the same, as any body would acquire in falling directly down from F to G . Now this is to be understood of the circular arch, which is described by the center of oscillation of the pendulum. I shall here farther observe, that if the straight line EG be drawn from the point, whence the pendulum falls, to the lowest point of the arch; in the same or in equal pendulums the velocity, which the

^a See Hugen. Horolog. Oscillat. p. 142.

pendulum acquires in G , is proportional to this line : that is, if the pendulum, after it has descended from E to G , be taken back to H , and let fall from thence, and the line HG be drawn ; the velocity, which the pendulum shall acquire in G by its descent from H , shall bear the same proportion to the velocity, which it acquires in falling from E to G , as the straight line HG bears to the straight line EG .

73. We may now proceed to those experiments upon the percussio of bodies, which I observed above might be made with pendulums. This expedient for examining the effects of percussio was first proposed by our late great architect Sir CHRISTOPHER WREN. And it is as follows. Two balls, as A and B (in fig. 59.) either equal or unequal, are hung by two strings from two points C and D , so that, when the balls hang down without motion, they shall just touch each other, and the strings be parallel. Here if one of these balls be removed to any distance from its perpendicular situation, and then let fall to descend and strike against the other ; by the last preceding paragraph it will be known, with what velocity this ball shall return into its first perpendicular situation, and consequently with what force it shall strike against the other ball ; and by the height to which this other ball ascends after the stroke, the velocity communicated to this ball will be discovered. For instance, let the ball A be taken up to E , and from thence be let fall to strike against B , passing over in its descent the circular arch EF . By this impulse let B fly up to G , moving through the circular arch HG . Then EI and GK being drawn horizontally, the

the ball A will strike against B with the velocity, which it would acquire in falling directly down from I ; and the ball B has received a velocity, wherewith, if it had been thrown directly upward, it would have ascended up to K. Likewise if straight lines be drawn from E to F and from H to G, the velocity of A, wherewith it strikes, will bear the same proportion to the velocity, which B has received by the blow, as the straight line EF bears to the straight line HG. In the same manner by noting the place to which A ascends after the stroke, its remaining velocity may be compared with that, wherewith it struck against B. Thus may be experimented the effects of the body A striking against B at rest. If both the bodies are lifted up, and so let fall as to meet and impinge against each other just upon the coming of both into their perpendicular situation ; by observing the places into which they move after the stroke, the effects of their percussion in all these cases may be found in the same manner as before.

74. Sir ISAAC NEWTON has described these experiments; and has shewn how to improve them to a greater exactness by making allowance for the resistance, which the air gives to the motion of the balls ^a. But as this resistance is exceeding small, and the manner of allowing for it is delivered by himself in very plain terms, I need not enlarge upon it here. I shall rather speak to a discovery, which he made by these experiments upon the elasticity of bodies. It has been explained above ^b, that when two bodies strike, if they be not elastic,

^a Princip. Philos. pag. 22.

^b Chap. 1. § 29.

they remain contiguous after the stroke; but that if they are elastic, they separate, and that the degree of their elasticity determines the proportion between the celerity wherewith they separate, and the celerity wherewith they meet. Now our author found, that the degree of elasticity appeared in the same bodies always the same, with whatever degree of force they struck; that is, the celerity wherewith they separated, always bore the same proportion to the celerity wherewith they met: so that the elastic power in all the bodies, he made trial upon, exerted it self in one constant proportion to the compressing force. Our author made trial with balls of wool bound up very compact, and found the celerity with which they receded, to bear about the proportion of 5 to 9 to the celerity wherewith they met; and in steel he found nearly the same proportion; in cork the elasticity was something less; but in glass much greater; for the celerity, wherewith balls of that material separated after percussion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met ^a.

75. I SHALL finish my discourse on pendulums, with this farther observation only, that the center of oscillation is also the center of another force. If a body be fixed to any point, and being put in motion turns round it; the body, if uninterrupted by the power of gravity or any other means, will continue perpetually to move about with the same equable motion. Now the force, with which such a body

^a Princip. Philos. pag. 25.

moves, is all united in the point, which in relation to the power of gravity is called the center of oscillation. Let the cylinder $ABCD$ (in fig. 60.) whose axis is EF , be fixed to the point E . And supposing the point E to be that on which the cylinder is suspended, let the center of oscillation be found in the axis EF , as has been explained above ^a. Let G be that center: then I say, that the force, wherewith this cylinder turns round the point E , is so united in the point G , that a sufficient force applied in that point shall stop the motion of the cylinder, in such a manner, that the cylinder should immediately remain without motion, though it were to be loosened from the point E at the same instant, that the impediment was applied to G : whereas, if this impediment had been applied to any other point of the axis, the cylinder would turn upon the point, where the impediment was applied. If the impediment had been applied between E and G , the cylinder would so turn on the point, where the impediment was applied, that the end BC would continue to move on the same way it moved before along with the whole cylinder; but if the impediment were applied to the axis farther off from E than G , the end AD of the cylinder would start out of its present place that way in which the cylinder moved. From this property of the center of oscillation, it is also called the center of percussion. That excellent mathematician, Dr. BROOK TAYLOR, has farther improved this doctrine concerning the center of percussion, by shewing, that if through this point G a line, as GHI , be drawn perpendicular to EF , and lying

^a § 71.

in the course of the body's motion; a sufficient power applied to any point of this line will have the same effect, as the like power applied to G^a : so that as we before shewed, the center of percussion within the body on its axis; by this means we may find this center on the surface of the body also, for it will be where this line HI crosses that surface.

76. I SHALL now proceed to the last kind of motion, to be treated on in this place, and shew what line the power of gravity will cause a body to describe, when it is thrown forwards by any force. This was first discovered by the great GALILEO, and is the principle, upon which engineers should direct the shot of great guns. But as in this case bodies describe in their motion one of those lines, which in geometry are called conic sections; it is necessary here to premise a description of those lines. In which I shall be the more particular, because the knowledge of them is not only necessary for the present purpose, but will be also required hereafter in some of the principal parts of this treatise.

77. THE first lines considered by the ancient geometers were the straight line and the circle. Of these they composed various figures, of which they demonstrated many properties, and resolved divers problems concerning them. These problems they attempted always to resolve by the describing straight lines and circles. For instance, let a square $ABCD$ (fig. 61.) be proposed, and let it be required to make ano-

^a See Method. Increment. prop. 25.

ther square in any assigned proportion to this. Prolong one side, as DA, of this square to E, till AE bear the same proportion to AD, as the new square is to bear to the square AC. If the opposite side BC of the square AC be also prolonged to F, till BF be equal to AE, and EF be afterwards drawn, I suppose my readers will easily conceive, that the figure ABFE will bear to the square ABCD the same proportion, as the line AE bears to the line AD. Therefore the figure ABFE will be equal to the new square, which is to be found, but is not itself a square, because the side AE is not of the same length with the side EF. But to find a square equal to the figure ABFE you must proceed thus. Divide the line DE into two equal parts in the point G, and to the center G with the interval GD describe the circle DHEI; then prolong the line AB, till it meets the circle in K; and make the square AKLM, which square will be equal to the figure ABFE, and bear to the square ABCD the same proportion, as the line AE bears to AD.

78. I SHALL not proceed to the proof of this, having only here set it down as a specimen of the method of resolving geometrical problems by the description of straight lines and circles. But there are some problems, which cannot be resolved by drawing straight lines or circles upon a plane. For the management therefore of these they took into consideration solid figures, and of the solid figures they found that, which is called a cone, to be the most useful.

79. A CONE is thus defined by EUCLIDE in his elements of geometry ¹. If to the straight line AB (in fig. 62,) another straight line, as AC , be drawn perpendicular, and the two extremities B and C be joined by a third straight line composing the triangle ACB (for so every figure is called, which is included under three straight lines:) then the two points A and B being held fixed, as two centers, and the triangle ACB being turned round upon the line AB , as on an axis; the line AC will describe a circle, and the figure ACB will describe a cone, of the form represented by the figure $BCDEF$ (fig. 63.) in which the circle $CDEF$ is usually called the base of the cone, and B the vertex.

80. Now by this figure may several problems be resolved, which cannot by the simple description of straight lines and circles upon a plane. Suppose for instance, it were required to make a cube, which should bear any assigned proportion to some other cube named. I need not here inform my readers, that a cube is the figure of a dye. This problem was much celebrated among the ancients, and was once enforced by the command of an oracle. This problem may be performed by a cone thus. First make a cone from a triangle, whose side AC shall be half the length of the side BC . Then on the plane $ABCD$ (fig. 64.) let the line EF be exhibited equal in length to the side of the cube proposed; and let the line FG be drawn perpendicular to EF , and of such a length, that it bear the same proportion to EF , as the

¹ Lib. XI. Def.

cube to be sought is required to bear to the cube proposed. Through the points E, F, and G let the circle FHI be described. Then let the line EF be prolonged beyond F to K, that FK be equal to FE, and let the triangle FKL, having all its sides FK, KL, LF equal to each other, be hung down perpendicularly from the plane ABCD. After this, let another plane MNOP be extended through the point L, so as to be equidistant from the former plane ABCD, and in this plane let the line QLR be drawn so, as to be equidistant from the line EFK. All this being thus prepared, let such a cone, as was above directed to be made, be so applied to the plane MNOP, that it touch this plane upon the line QR, and that the vertex of the cone be applied to the point L. This cone, by cutting through the first plane ABCD, will cross the circle FHI before described. And if from the point S, where the surface of this cone intersects the circle, the line ST be drawn so, as to be equidistant from the line EF; the line FT will be equal to the side of the cube sought : that is, if there be two cubes or dyes formed, the side of one being equal to EF, and the side of the other equal to FT; the former of these cubes shall bear the same proportion to the latter, as the line EF bears to FG.

81. INDEED this placing a cone to cut through a plane is not a practicable method of resolving problems. But when the geometers had discovered this use of the cone, they applied themselves to consider the nature of the lines, which will be produced by the intersection of the surface of a cone

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and a plane ; whereby they might be enabled both to reduce these kinds of solutions to practice, and also to render their demonstrations concise and elegant.

82. WHENEVER the plane, which cuts the cone, is equidistant from another plane, that touches the cone on the side; (which is the case of the present figure ;) the line, wherein the plane cuts the surface of the cone, is called a parabola. But if the plane, which cuts the cone, be so inclined to this other, that it will pass quite through the cone (as in fig. 65.) such a plane by cutting the cone produces the figure called an ellipsis, in which we shall hereafter shew the earth and other planets to move round the sun. If the plane, which cuts the cone, recline the other way (as in fig. 66.) so as not to be parallel to any plane, whereon the cone can lie, nor yet to cut quite through the cone ; such a plane shall produce in the cone a third kind of line, which is called an hyperbola. But it is the first of these lines named the parabola, wherein bodies, that are thrown obliquely, will be carried by the force of gravity ; as I shall here proceed to shew, after having first directed my readers how to describe this sort of line upon a plane, by which the form of it may be seen.

83. To any straight line AB (fig. 67.) let a straight ruler CD be so applied, as to stand against it perpendicularly. Upon the edge of this ruler let another ruler EF be so placed, as to move along upon the edge of the first ruler CD, and keep always perpendicular to it. This being so disposed, let any point, as G, be taken in the line AB, and let a string equal
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in length to the ruler EF be fastened by one end to the point G, and by the other to the extremity F of the ruler EF. Then if the string be held down to the ruler EF by a pin H, as is represented in the figure; the point of this pin, while the ruler EF moves on the ruler CD, shall describe the line IKL, which will be one part of the curve line, whose description we were here to teach: and by applying the rulers in the like manner on the other side of the line AB, we may describe the other part IM of this line. If the distance CG be equal to half the line EF in fig. 64, the line MIL will be that very line, wherein the plane ABCD in that figure cuts the cone.

84. THE line AI is called the axis of the parabola MIL, and the point G is called the focus.

85. Now by comparing the effects of gravity upon falling bodies, with what is demonstrated of this figure by the geometers, it is proved, that every body thrown obliquely is carried forward in one of these lines, the axis whereof is perpendicular to the horizon.

86. THE geometers demonstrate, that if a line be drawn to touch a parabola in any point, as the line AB (in fig. 68.) touches the parabola CD, whose axis is YZ, in the point E; and several lines FG, HI, KL be drawn parallel to the axis of the parabola: then the line FG will be to HI in the duplicate proportion of EF to EH, and FG to KL in the duplicate proportion of EF to EK; likewise HI to KL in the duplicate proportion of EH to EK. What is to be understood by duplicate or two-fold

proportion, has been already explained ^a. Accordingly I mean here, that if the line M be taken to bear the same proportion to EH, as EH bears to EF, HI will bear the same proportion to FG, as M bears to EF; and if the line N bears the same proportion to EK, as EK bears to EF, KL will bear the same proportion to FG, as N bears to EF; or if the line O bear the same proportion to EK, as EK bears to EH, KL will bear the same proportion to HI, as O bears to EH.

87. THIS property is essential to the parabola, being so connected with the nature of the figure, that every line possessing this property is to be called by this name.

88. Now suppose a body to be thrown from the point A (in fig. 69.) towards B in the direction of the line AB. This body, if left to itself, would move on with a uniform motion through this line AB. Suppose the eye of a spectator to be placed at the point C just under the point A; and let us imagine the earth to be so put into motion along with the body, as to carry the spectator's eye along the line CD parallel to AB; and that the eye should move on with the same velocity, wherewith the body would proceed in the line AB, if it were to be left to move without any disturbance from its gravitation towards the earth. In this case if the body moved on without being drawn towards the earth, it would appear to the spectator to be at rest. But if the power of gravity exerted itself on the body, it would appear to the spe-

^a Chap. 2. § 17.

etator to fall directly down. Suppose at the distance of time, wherein the body by its own progressive motion would have moved from A to E, it should appear to the spectator to have fallen through a length equal to EF: then the body at the end of this time will actually have arrived at the point F. If in the space of time, wherein the body would have moved by its progressive motion from A to G, it would have appeared to the spectator to have fallen down the space GH: then the body at the end of this greater interval of time will be arrived at the point H. Now if the line AFHI be that, through which the body actually passes; from what has here been said, it will follow, that this line is one of those, which I have been describing under the name of the parabola. For the distances EF, GH, through which the body is seen to fall, will increase in the duplicate proportion of the times^a; but the lines AE, AG will be proportional to the times wherein they would have been described by the single progressive motion of the body: therefore the lines EF, GH will be in the duplicate proportion of the lines AF, AG; and the line AFHI possesses the property of the parabola.

89. IF the earth be not supposed to move along with the body, the case will be a little different. For the body being constantly drawn directly towards the center of the earth, the body in its motion will be drawn in a direction a little oblique to that, wherein it would be drawn by the earth in motion, as before supposed. But the distance to the center of the

^a See above Ch. 2. § 17.

earth bears so vast a proportion to the greatest length, to which we can throw bodies, that this obliquity does not merit any regard. From the sequel of this discourse it may indeed be collected, what line the body being thrown thus would be found to describe, allowance being made for this obliquity of the earth's action^a. This is the discovery of Sir IS. NEWTON; but has no use in this place. Here it is abundantly sufficient to consider the body as moving in a parabola.

90. THE line, which a projected body describes, being thus known, practical methods have been deduced from hence for directing the shot of great guns to strike any object desired. This work was first attempted by GALILEO, and soon after farther improved by his scholar TORRICELLI; but has lately been rendered more complete by the great Mr. COTES, whose immature death is an unspeakable loss to mathematical learning. If it be required to throw a body from the point A (in fig. 70.) so as to strike the point B; through the points A, B draw the straight line CD, and erect the line AE perpendicular to the horizon, and of four times the height, from which a body must fall to acquire the velocity, wherewith the body is intended to be thrown. Through the points A and E describe a circle, that shall touch the line CD in the point A. Then from the point B draw the line BF perpendicular to the horizon, intersecting the circle in the points G and H. This being done, if the body be projected directly towards either of these points G or H, it shall fall upon the point B; but with this difference, that, if it be thrown

^a From B. II. Ch. 3.

in the direction AG , it shall sooner arrive at B , than if it were projected in the direction AH . When the body is projected in the direction AG ; the time, it will take up in arriving at B , will bear the same proportion to the time, wherein it would fall down through one fourth part of AE , as AG bears to half AE . But when the body is thrown in the direction of AH , the time of its passing to B will bear the same proportion to the time, wherein it would fall through one fourth part of AE , as AH bears to half AE .

91. If the line AI be drawn so as to divide the angle under EAD in the middle, and the line IK be drawn perpendicular to the horizon; this line will touch the circle in the point I , and if the body be thrown in the direction AI , it will fall upon the point K : and this point K is the farthest point in the line AD , which the body can be made to strike, without increasing its velocity.

92. THE velocity, wherewith the body every where moves, may be found thus. Suppose the body to move in the parabola AB (fig. 71.) Erect AC perpendicular to the horizon, and equal to the height, from which a body must fall to acquire the velocity, wherewith the body sets out from A . If you take any points as D and E in the parabola, and draw DF and EG parallel to the horizon; the velocity of the body in D will be equal to what a body will acquire in falling down by its own weight through CF , and in E the velocity will be the same, as would be acquired in falling through CG . Thus the body moves slowest at the highest point H of the parabola; and at equal distances from this point will move

move with equal swiftness, and descend from that highest point through the line HB altogether like to the line AH in which it ascended; abating only the resistance of the air, which is not here considered. If the line HI be drawn from the highest point H parallel to the horizon, AI will be equal to $\frac{1}{4}$ of BG in fig. 70, when the body is projected in the direction AG, and equal to $\frac{1}{4}$ of BH, when the body is thrown in the direction AH provided AD be drawn horizontally.

93. THUS I have recounted the principal discoveries, which had been made concerning the motion of bodies by Sir ISAAC NEWTON's predecessors; all these discoveries, by being found to agree with experience, contributing to establish the laws of motion, from whence they were deduced. I shall therefore here finish what I had to say upon those laws; and conclude this chapter with a few words concerning the distinction which ought to be made between absolute and relative motion. For some have thought fit to confound them together; because they observe the laws of motion to take place here on the earth, which is in motion, after the same manner as if it were at rest. But Sir ISAAC NEWTON has been careful to distinguish between the relative and absolute consideration both of motion and time^a. The astronomers anciently found it necessary to make this distinction in time. Time considered in it self passes on equably without relation to any thing external, being the proper measure of the continuance and duration of all things. But it is most frequently conceived of by us under a relative view to some succession in

^a Prin. Philos. pag. 7, &c.

sensible things, of which we take cognizance. The succession of the thoughts in our own minds is that, from whence we receive our first idea of time, but is a very uncertain measure thereof; for the thoughts of some men flow on much more swiftly, than the thoughts of others; nor does the same person think equally quick at all times. The motions of the heavenly bodies are more regular; and the eminent division of time into night and day, made by the sun, leads us to measure our time by the motion of that luminary: nor do we in the affairs of life concern our selves with any inequality, which there may be in that motion; but the space of time which comprehends a day and night is rather supposed to be always the same. However astronomers anciently found these spaces of time not to be always of the same length, and have taught how to compute their differences. Now the ~~time, when~~ so equated as to be rendered perfectly equal, is the true measure of duration, the other not. And therefore this latter, which is absolutely true time, differs from the other, which is only apparent. And as we ordinarily make no distinction between apparent time, as measured by the sun, and the true; so we often do not distinguish in our usual discourse between the real, and the apparent or relative motion of bodies; but use the same words for one, as we should for the other. Though all things about us are really in motion with the earth; as this motion is not visible, we speak of the motion of every thing we see, as if our selves and the earth stood still. And even in other cases, where we discern the motion of bodies, we often speak of them not in relation to the whole motion we see, but with regard to other

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bodies, to which they are contiguous. If any body were lying on a table ; when that table shall be carried along, we say the body rests upon the table, or perhaps absolutely, that the body is at rest. However philosophers must not reject all distinction between true and apparent motions, any more than astronomers do the distinction between true and vulgar time ; for there is as real a difference between them, as will appear by the following consideration. Suppose all the bodies of the universe to have their courses stopped, and reduced to perfect rest. Then suppose their present motions to be again restored ; this cannot be done without an actual impression made upon some of them at least. If any of them be left untouched, they will retain their former state, that is, still remain at rest ; but the other bodies, which are wrought upon, will have changed their former state of rest, for the contrary state of motion. Let us now suppose the bodies left at rest to be annihilated, this ~~will~~ make no alteration in the state of the moving bodies ; but the effect of the impression, which was made upon them, will still subsist. This shews the motion they received to be an absolute thing, and to have no necessary dependence upon the relation which the body said to be in motion has to any other body ^a.

94. BESIDES absolute and relative motion are distinguishable by their Effects. One effect of motion is, that bodies, when moved round any center or axis, acquire a certain

^a See Newton. princip. philos. pag. 9. lin. 30.

power,

power, by which they forcibly press themselves from that center or axis of motion. As when a body is whirled about in a sling, the body presses against the sling, and is ready to fly out as soon as liberty is given it. And this power is proportional to the true, not relative motion of the body round such a center or axis. Of this Sir ISAAC NEWTON gives the following instance ^a. If a pail or such like vessel near full of water be suspended by a string of sufficient length, and be turned about till the string be hard twisted. If then as soon as the vessel and water in it are become still and at rest, the vessel be nimbly turned about the contrary way the string was twisted, the vessel by the strings untwisting it self shall continue its motion a long time. And when the vessel first begins to turn, the water in it shall receive little or nothing of the motion of the vessel, but by degrees shall receive a communication of motion, till at last it shall move round as swiftly as the vessel it self. Now the definition of motion, which DES CARTES has given us upon this principle of making all motion meerly relative, is this : that motion, is a removal of any body from its vicinity to other bodies, which were in immediate contact with it, and are considered as at rest ^b. And if this be compared with what he soon after says, that there is nothing real or positive in the body moved, for the sake of which we ascribe motion to it, which is not to be found as well in the contiguous bodies, which are considered as at rest ^c; it will follow from thence, that we may consider the vessel as at rest

^a Princip. Philos. pag. 10.

^b Renat. Des Cart. Princ. Philos. Part. II. § 25.

^c Ibid. § 30.

and the water as moving in it : and the water in respect of the vessel has the greatest motion, when the vessel first begins to turn, and loses this relative motion more and more, till at length it quite ceases. But now, when the vessel first begins to turn, the surface of the water remains smooth and flat, as before the vessel began to move ; but as the motion of the vessel communicates by degrees motion to the water, the surface of the water will be observed to change, the water subsiding in the middle and rising at the edges : which elevation of the water is caused by the parts of it pressing from the axis, they move about ; and therefore this force of receding from the axis of motion depends not upon the relative motion of the water within the vessel, but on its absolute motion ; for it is least, when that relative motion is greatest, and greatest, when that relative motion is least, or none at all.

95. THUS the true cause of what appears in the surface of this water cannot be assigned, without considering the water's motion within the vessel. So also in the system of the world, in order to find out the cause of the planetary motions, we must know more of the real motions, which belong to each planet, than is absolutely necessary for the uses of astronomy. If the astronomer should suppose the earth to stand still, he could ascribe such motions to the celestial bodies, as should answer all the appearances ; though he would not account for them in so simple a manner, as by attributing motion to the earth. But the motion of the earth must of necessity be considered, before the real causes, which actuate the planetary system, can be discovered.

CHAP.

CHAP. III.

OF CENTRIPETAL FORCES.

WE have just been describing in the preceding chapter the effects produced on a body in motion, from its being continually acted upon by a power always equal in strength, and operating in parallel directions^a. But bodies may be acted upon by powers, which in different places shall have different degrees of force, and whose several directions shall be variously inclined to each other. The most simple of these in respect to direction is, when the power is pointed constantly to one center. This is truly the case of that power, whose effects we described in the foregoing chapter; though the center of that power is so far removed, that the subject then before us is most conveniently to be considered in the light, wherein we have placed it: But Sir ISAAC NEWTON has considered very particularly this other case of powers, which are constantly directed to the same center. It is upon this foundation, that all his discoveries in the system of the world are raised. And therefore, as this subject bears so very great a share in the philosophy, of which I am discoursing, I think it proper in this place to take a short view of some of the general effects of these powers, before we come to apply them particularly to the system of the world.

^a § 85, &c.

2. THESE powers or forces are by Sir ISAAC NEWTON called centripetal; and their first effect is to cause the body, on which they act, to quit the straight course, wherein it would proceed if undisturbed, and to describe an incurvated line, which shall always be bent towards the center of the force. It is not necessary, that such a power should cause the body to approach that center. The body may continue to recede from the center of the power, notwithstanding its being drawn by the power; but this property must always belong to its motion, that the line, in which it moves, will continually be concave towards the center, to which the power is directed. Suppose A (in fig. 72.) to be the center of a force. Let a body in B be moving in the direction of the straight line BC, in which line it would continue to move, if undisturbed; but being attracted by the centripetal force towards A, ~~the body~~ must necessarily depart from this line BC, ~~and~~ being drawn into the curve line BD, must pass between the lines AB and BC. It is evident therefore, that the body in B being gradually turned off from the straight line BC, it will at first be convex toward the line BC, and consequently concave towards the point A: for these centripetal powers are supposed to be in strength proportional to the power of gravity, and, like that, not to be able after the manner of an impulse to turn the body sensibly out of its course into a different one in an instant, but to take up some space of time in producing a visible effect. That the curve will always continue to have its concavity towards A may thus appear. In the line BC near to B take any point as E, from which the line EFG may be so drawn

drawn, as to touch the curve line BD in some point as F. Now when the body is come to F, if the centripetal power were immediately to be suspended, the body would no longer continue to move in a curve line, but being left to it self would forthwith reassume a straight course; and that straight course would be in the line FG: for that line is in the direction of the body's motion at the point F. But the centripetal force continuing its energy, the body will be gradually drawn from this line FG so as to keep in the line FD, and make that line near the point F to be convex toward FG, and concave toward A. After the same manner the body may be followed on in its course through the line BD, and every part of that line be shewn to be concave toward the point A.

3. THIS then is the constant character belonging to those motions, which are carried on by centripetal forces; that the line, wherein the body moves, is throughout concave towards the center of the force. In respect to the successive distances of the body from the center there is no general rule to be laid down; for the distance of the body from the center may either increase, or decrease, or even keep always the same. The point A (in fig. 73.) being the center of a centripetal force, let a body at B set out in the direction of the straight line BC perpendicular to the line AB drawn from A to B. It will be easily conceived, that there is no other point in the line BC so near to A, as the point B; that AB is the shortest of all the lines, which can be drawn from A to any part of the line BC; all other lines, as AD, or AE, drawn from A to the line BC being longer than AB. Hence it follows, that the body setting

ting out from B, if it moved in the line BC, it would recede more and more from the point A. Now as the operation of a centripetal force is to draw a body towards the center of the force: if such a force act upon a resting body, it must necessarily put that body so into motion, as to cause it to move towards the center of the force: if the body were of it self moving towards that center, the centripetal force would accelerate that motion, and cause it to move faster down: but if the body were in such a motion, as being left to itself it would recede from this center, it is not necessary, that the action of a centripetal power upon it should immediately compel the body to approach the center, from which it would otherwise have receded; the centripetal power is not without effect, if it cause the body to recede more slowly from that center, than otherwise it would have done. Thus in the case before us, the smallest centripetal power, if it act on the body, will force it out of the line BC, and cause it to pass in a bent line between BC and the point A, as has been before explained. When the body, for instance, has advanced to the line AD, the effect of the centripetal force discovers it self by having removed the body out of the line BC, and brought it to cross the line AD somewhere between A and D: suppose at F. Now AD being longer than AB, AF may also be longer than AB. The centripetal power may indeed be so strong, that AF shall be shorter than AB; or it may be so evenly balanced with the progressive motion of the body, that AF and AB shall be just equal: and in this last case, when the centripetal force is of that strength, as constantly to draw the body as much toward
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the center, as the progressive motion would carry it off, the body will describe a circle about the center A, this center of the force being also the center of the circle.

4. IF the body, instead of setting out in the line BC perpendicular to AB, had set out in another line BG more inclined towards the line ~~AB~~, moving in the curve line BH; then as the body, if it were to continue its motion in the line BG, would for some time approach the center A; the centripetal force would cause it to make greater advances toward that center. But if the body were to set out in the line BI reclined the other way from the perpendicular BC, and were to be drawn by the centripetal force into the curve line BK; the body, notwithstanding any centripetal force, would for some time recede from the center; since some part at least of the curve line ~~BK lies between~~ the line BI and the perpendicular BC.

5. THUS far we have explained such effects, as attend every centripetal force. But as these forces may be very different in regard to the different degrees of strength, where-with they act upon bodies in different places; I shall now proceed to make mention in general of some of the differences attending these centripetal motions.

6. TO reassume the consideration of the last mentioned case. Suppose a centripetal power directed toward the point A (in fig. 74.) to act on a body in B, which is moving in the direction of the straight line BC, the line BC reclining off from AB. If from A the straight lines AD, AE, AF are

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drawn

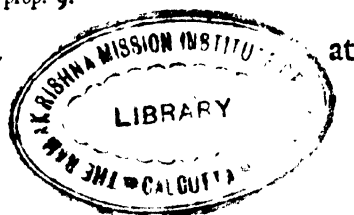
drawn at pleasure to the line CB; the line CB being prolonged beyond B to G, it appears that AD is inclined to the line GC more obliquely, than AB is inclined to it, AE is inclined more obliquely than AD, and AF more than AE. To speak more correctly, the angle under ADG is less than that under ABG, the angle under AEG less than that under ADG, and the angle under AFG less than that under AEG. Now suppose the body to move in the curve line BHIK. Then it is here likewise evident, that the line BHIK being concave towards A, and convex towards the line BC, it is more and more turned off from the line BC; so that in the point H the line AH will be less obliquely inclined to the curve line BHIK, than the same line AHD is inclined to BC at the point D; at the point I the inclination of the line AI to the curve line will be more different from the inclination of the same line AIE to the line BC, at the point E; and in the points K and F the difference of inclination will be still greater; and in both the inclination at the curve will be less oblique, than at the straight line BC. But the straight line AB is less obliquely inclined to BG, than AD is inclined towards DG: therefore although the line AH be less obliquely inclined towards the curve HB, than the same line AHD is inclined towards DG; yet it is possible, that the inclination at H may be more oblique, than the inclination at B. The inclination at H may indeed be less oblique than the other, or they may be both the same. This depends upon the degree of strength, wherewith the centripetal force exerts it self, during the passage of the body from B to H. After the same manner the inclinations at I and K depend entirely on the degree

gree of strength, wherewith the centripetal force acts on the body in its passage from H to K: if the centripetal force be weak enough, the lines AH and AI drawn from the center A to the body at H and at I shall be more obliquely inclined to the curve, than the line AB is inclined towards BG. The centripetal force may be of that strength as to render all these inclinations equal, ~~or if stronger~~, the inclinations at I and K will be less oblique than at B. Sir ISAAC NEWTON has particularly shewn, that if the centripetal power decreases after a certain manner with the increase of distance, a body may describe such a curve line, that all the lines drawn from the center to the body shall be equally inclined to that curve line^a. But I do not here enter into any particulars, my present intention being only to shew, that it is possible for a body to be acted upon by a force continually drawing it down towards a center, ~~and yet that the body shall continue to recede from that center~~; for here as long as the lines AH, AI, &c drawn from the center A to the body do not become less oblique to the curve, in which the body moves; so long shall those lines perpetually increase, and consequently the body shall more and more recede from the center.

7. BUT we may observe farther, that if the centripetal power, while the body increases its distance from the center, retain sufficient strength to make the lines drawn from the center to the body to become at length less oblique to the curve; then if this diminution of the obliquity continue, till

^a Princip. Philos. Lib. I. prop. 9.

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at last the line drawn from the center to the body shall cease to be obliquely inclined to the curve, and shall become perpendicular thereto; from this instant the body shall no longer recede from the center, but in its following motion it shall again descend, and shall describe a curve line in all respects like to that, which it has described already; provided the centripetal power, every where at the same distance from the center, acts with the same strength. So we observed in the preceding chapter, that, when the motion of a projectile became parallel to the horizon, the projectile no longer ascended, but forthwith directed its course downwards, descending in a line altogether like that, wherein it had before ascended^a.

8. THIS return of the body may be proved by the following proposition: that if the body in any place, suppose at I, were to be stopt, and be thrown directly backward with the velocity, wherewith it was moving forward in that point I; then the body, by the action of the centripetal force upon it, would move back again over the path I H B, in which it had before advanced forward, and would arrive again at the point B in the same space of time, as was taken up in its passage from B to I; the velocity of the body at its return to the point B being the same, as that wherewith it first set out from that point. To give a full demonstration of this proposition, would require that use of mathematics, which I here purpose to avoid; but, I believe, it will appear in great measure evident from the following considerations.

^a § 92.

9. SUPPOSE (in fig. 75.) that a body were carried after the following manner through the bent figure ABCDEF, composed of the straight lines AB, BC, CD, DE, EF. First let it be moving in the line AB, from A towards B, with any uniform velocity. At B let the body receive an impulse directed toward some point, as G, taken within the concavity of the figure. ~~Now whereas~~ this body, when once moving in the straight line AB, will continue to move on in this line, so long as it shall be left to it self; but being disturbed at the point B in its motion by the impulse, which there acts upon it, it will be turned out of this line AB into some other straight line, wherein it will afterwards continue to move, as long as it shall be left to itself. Therefore let this impulse have strength sufficient to turn the body into the line BC. Then let the body move on undisturbed from B to C, but at C let it receive another impulse pointed toward the same point G, and of sufficient strength to turn the body into the line CD. At D let a third impulse, directed like the rest to the point G, turn the body into the line DE. And at E let another impulse, directed likewise to the point G, turn the body into the line EF. Now, I say, if the body while moving in the line EF be stopt, and turned back again in this line with the same velocity, as that wherewith it was moving forward in this line; then by the repetition of the former impulse at E the body will be turned into the line ED, and move in it from E to D with the same velocity as before it moved with from D to E; by the repetition of the impulse at D, when the body shall have returned to that point, it will be turned into the line DC; and by the repetition of the other impulses at C and B.

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the body will be brought back again into the line BA , with the velocity, wherewith it first moved in that line.

10. THIS I prove as follows. Let DE and FE be continued beyond E . In DE thus continued take at pleasure the length EH , and let HI be so drawn, as to be equidistant from the line GE . Then, by what has been written upon the second law of motion ^a, it follows, that after the impulse on the body in E it will move through EI in the same time, as it would have imployed in moving from E to H , with the velocity which it had in the line DE . In FE prolonged take EK equal to EI , and draw KL equidistant from GE . Then, because the body is thrown back in the line FE with the same velocity as that wherewith it went forward in that line; if, when the body was returned to E , it were permitted to go straight on, it would pass through EK in the same time, as it took up in passing through EI , when it went forward in the line EF . But, if at the body's return to the point E , such an impulse directed toward the point D were to be given it, whereby it should be turned into the line DE ; I say, that the impulse necessary to produce this effect must be equal to that, which turned the body out of the line DE into EF ; and that the velocity, with which the body will return into the line ED , is the same, as that wherewith it before moved through this line from D to E . Because EK is equal to EI , and KL and HI , being each equidistant from GE , are by consequence equidistant from each other; it follows, that the two

triangular figures IEH and KEL are altogether like and equal to each other. If I were writing to mathematicians, I might refer them to some propositions in the elements of EUCLID for the proof of this^a: but as I do not here address my self to such, so I think this assertion will be evident enough without a proof in form; at least I must desire my readers to receive it as a proposition true in geometry. But these two triangular figures being altogether like each other and equal; as EK is equal to EI , so EL is equal to EH , and KL equal to HI . Now the body after its return to E being turned out of the line FE into ED by an impulse acting upon it in E , after the manner above expressed; the body will receive such a velocity by this impulse, as will carry it through EL in the same time, as it would have imployed in passing through EK , if it had gone on in that line undisturbed. And it has already been observed, that the time, in which the body would pass over EK with the velocity wherewith it returns, is equal to the time it took up in going forward from E to I ; that is, equal to the time, in which it would have gone through EH with the velocity, wherewith it moved from D to E . Therefore the time, in which the body will pass through EL after its return into the line ED , is the same, as would have been taken up by the body in passing through EH with the velocity, wherewith the body first moved in the line DE . Since therefore EL and EH are equal, the body returns into the line DE with the velocity, which it had before in that line. Again I say, the second impulse in E is equal to the first. By what has

^a Viz. L. I. prop. 30, 29, & 26.

been

been said on the second law of motion concerning the effect of oblique impulses^a, it will be understood, that the impulse in B, whereby the body was turned out of the line DE into the line EF, is of such strength, that if the body had been at rest, when this impulse had acted upon it, this impulse would have communicated so much motion to the body, as would have carried it through a length equal to HI , in the time wherein the body would have passed from E to H, or in the time wherein it passed from E to I. In the same manner, on the return of the body, the impulse in E, whereby the body is turned out of the line FE into ED, is of such strength, that if it had acted on the body at rest, it would have caused the body to move through a length equal to KL , in the same time, as the body would employ in passing through EK with the velocity, wherewith it returns in the line FE. Therefore the second impulse, had it acted on the body at rest, would have caused it to move through a length equal to KL in the same space of time, as would be taken up by the body in passing through a length equal to HI , were the first impulse to act on the body when at rest. That is, the effects of the first and second impulse on the body when at rest would be the same; for KI and HI are equal: consequently the second impulse is equal to the first.

II. THUS if the body be returned through FE with the velocity, wherewith it moved forward; we have shewn how by the repetition of the impulse, which acted on it at E, the

^a Ch. II. § 21, 22.

body will return again into the line DE with the velocity, which it had before in that line. By the same process of reasoning it may be proved, that, when the body is returned back to D, the impulse, which before acted on the body at that point, will throw the body into the line DC with the velocity, which it first had in that line; and the other impulses being successively repeated, the body will at length be brought back again into the line BA with the velocity, wherewith it set out in that line.

12. THUS these impulses, by acting over again in an inverted order all their operation on the body, bring it back again through the path, in which it had proceeded forward. And this obtains equally, whatever be the number of the straight lines, whereof this curve figure is composed. Now by a method of reasoning, which Sir ISAAC NEWTON makes great use of, and which he introduced into geometry, thereby greatly enriching that science^a; we might make a transition from this figure composed of a number of straight lines to a figure of one continued curvature, and from a number of separate impulses repeated at distinct intervals to a continual centripetal force, and shew, that, because what has been here advanced holds universally true, whatever be the number of straight lines, whereof the curve figure ACF is composed, and howsoever frequently the impulses at the angles of this figure are repeated; therefore the same will still remain true, although this figure should be converted into one of a continued curvature, and these distinct impulses should be

^a viz. His doctrine of prime and ultimate ratios.

changed into a continual centripetal force. But as the explaining this method of reasoning is foreign to my present design; so I hope my readers, after what has been said, will find ~~no~~ difficulty in receiving the proposition laid down above: that, if the body, which has moved through the curve line BHI (in fig. 74.) from B to I, when it is come to I, be thrown directly back with the same velocity as that, ~~where~~ ~~it~~ ~~had~~ ~~proceeded~~ forward, the centripetal force, by acting over again all its operation on the body, shall bring the body back again in the line IHB: and as the motion of the body in its course from B to I was every where in such a manner oblique to the line drawn from the center to the body, that the centripetal power acted in some degree against the body's motion, and gradually diminished it; so in the return of the body, the centripetal power will every where draw the body forward, and accelerate its motion by the same degrees, as before it retarded it.

13. THIS being agreed, suppose the body in K to have the line AK no longer obliquely inclined to its motion. In this case, if the body be turned back, in the manner we have been considering, it must be directed back perpendicularly to AK. But if it had proceeded forward, it would likewise have moved in a direction perpendicular to AK; consequently, whether it move from this point K backward or forward, it must describe the same kind of course. Therefore since by being turned back it will go over again the line KIH B; if it be permitted to go forward, the line KL, which it shall describe, will be altogether similar to the line KHB.

14. IN like manner we may determine the nature of the motion, if the line, wherein the body sets out, be inclined (as in fig. 76.) down toward the line BA drawn between the body and the center. If the centripetal power so much increases in strength, as the body approaches, that it can bend the path, in which the body moves, to that degree, as to cause all the lines as ~~AI, AI, AK~~ to remain no less oblique to the motion of the body, than AB is oblique to BC; the body shall continually more and more approach the center. But if the centripetal power increases in so much less a degree, as to permit the line drawn from the center to the body, as it accompanies the body in its motion, at length to become more and more erect to the curve wherein the body moves, and in the end, suppose at K, to become perpendicular thereto; from that time the body shall rise again. This is evident from what has been said above; because for the very same reason here also the body shall proceed from the point K to describe a line altogether similar to the line, in which it has moved from B to K. Thus, as it was observed of the pendulum in the preceding chapter^a, that all the time it approaches towards being perpendicular to the horizon, it more and more descends; but, as soon as it is come into that perpendicular situation, it immediately rises again by the same degrees, as it descended by before: so here the body more and more approaches the center all the time it is moving from B to K; but thence forward it rises from the center again by the same degrees, as it approached by before.

15. IF (in fig. 77.) the line BC be perpendicular to AB ; then it has been observed above, that the centripetal power may be so balanced with the progressive motion of the body, that the body may keep moving round the center A constantly at the same distance; as a body does, when whirled about any point, to which it is tied by a string. If the centripetal power be too weak to produce this effect, ~~the motion~~ of the body will presently become oblique to the line drawn from itself to the center, after the manner of the first of the two cases, which we have been considering. If the centripetal power be stronger, than what is required to carry the body in a circle, the motion of the body will presently fall in with the second of the cases, we have been considering.

16. IF the centripetal power so change with the change of distance, that the body, after its motion has become oblique to the line drawn from itself to the center, shall again become perpendicular thereto; which we have shewn to be possible in both the cases treated of above; then the body shall in its subsequent motion return again to the distance of AB , and from that distance take a course similar to the former: and thus, if the body move in a space free from all resistance, which has been here all along supposed; it shall continue in a perpetual motion about the center, descending and ascending alternately therefrom. If the body setting out from B (in fig. 78.) in the line BC perpendicular to AB , describe the line BDE , which in D shall be oblique to the line AD , but in E

shall again become erect to AE drawn from the body in E to the center A; then from this point E the body shall describe the line EFG altogether like to the line BDE, and at G shall be at the same distance from A, as it was at B. But likewise the line AG shall be erect to the body's motion. Therefore the body shall proceed to describe from G the line GHI altogether similar to the line GFE, and at I have the same distance from the center, as it had at E; and also have the line AI erect to its motion: so that its following motion must be in the line IKL similar to IHG, and the distance AL equal to AG. Thus the body will go on in a perpetual round without ceasing, alternately enlarging and contracting its distance from the center.

17. IF it so happen, that the point E fall upon the line BA continued beyond A; then the point G will fall on B, I on E, and L also on B; so that the body will describe in this case a simple curve line round the center A, like the line BDEF in fig. 79, in which it will continually revolve from B to E and from E to B without end.

18. IF AE in fig. 78 should happen to be perpendicular to AB, in this case also a simple line will be described; for the point G will fall on the line BA prolonged beyond A, the point I on the line AE prolonged beyond A, and the point L on B: so that the body will describe a line like the curve line BEGI in fig. 80, in which the opposite points B and G are equally distant from A, and the opposite points E and I are also equally distant from the same point A.

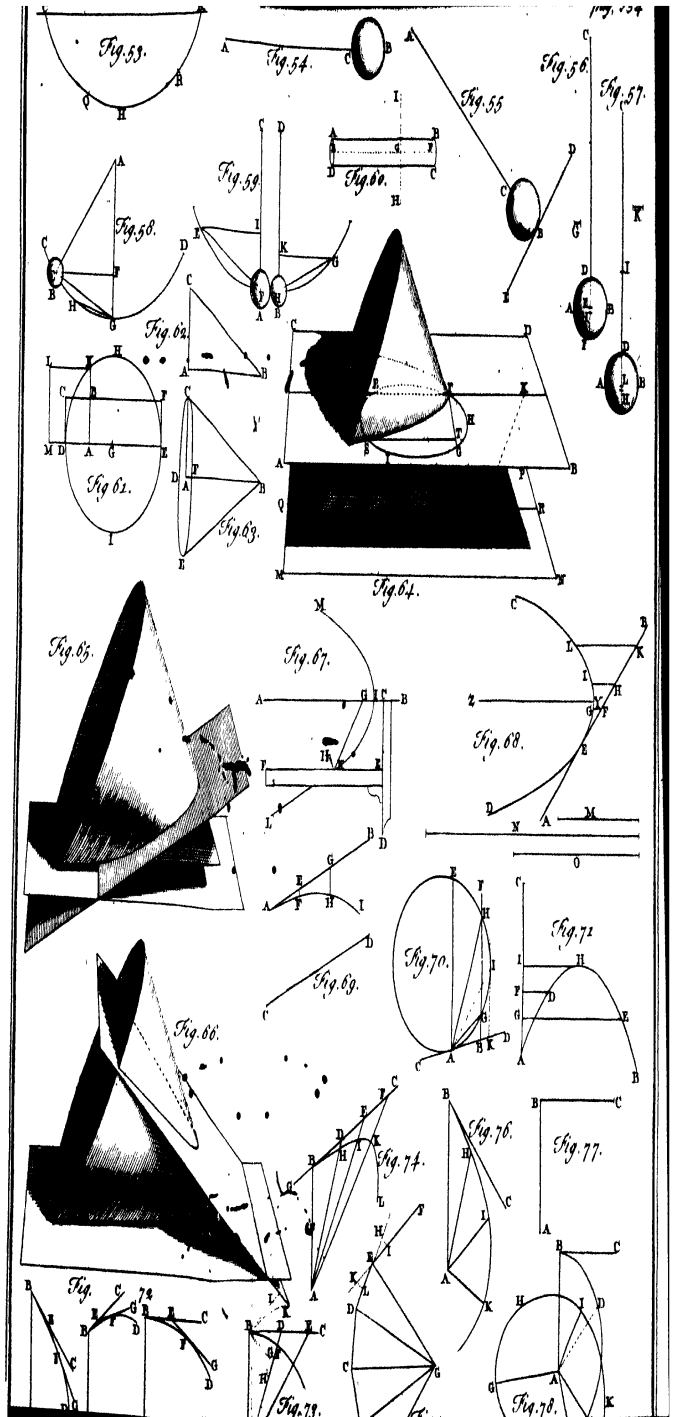
19. IN other cases the line described will have a more complex figure.

20. THUS we have endeavoured to shew how a body, while it is constantly attracted towards a center, may notwithstanding by its progressive motion keep it self from falling down to that center; but describe ~~about it an~~ endless circuit, sometimes approaching toward that center, and at other times as much receding from the same.

21. BUT here we have supposed, that the centripetal power is of equal strength every where at the same distance from the center. And this is the case of that centripetal power, which will hereafter be shewn to be the cause, that keeps the planets in their courses. But a body may be kept on in a perpetual circuit round a center, although the centripetal power have not this property. Indeed a body may by a centripetal force be kept moving in any curve line whatever, that shall have its concavity turned every where towards the center of the force.

22. TO make this evident I shall first propose the case of a body moving through the incurvated figure ABCDE (in fig. 81.) which is composed of the straight lines AB, BC, CD, DE, and EA; the motion being carried on in the following manner. Let the body first move in the line AB with any uniform velocity. When it is arrived at the point B, let it receive an impulse directed toward any point F taken within the figure; and let the impulse be of that strength as to turn the body out

of



of the line AB into the line BC. The body after this impulse, while left to itself, will continue moving in the line BC. At C let the body receive another impulse directed towards the same point F, of such strength, as to turn the body from the line BC into the line CD. At D let the body by another impulse, directed likewise to the point F, be turned out of the line CD into DE. And at E let another impulse, directed toward the point F, turn the body from the line DE into EA. Thus we see how a body may be carried through the figure ABCDE by certain impulses directed always toward the same center, only by their acting on the body at proper intervals, and with due degrees of strength.

23. BUT farther, when the body is come to the point A, if it there receive another impulse directed like the rest toward the point F, and of such a degree of strength as to turn the body into the line AF, wherein it first moved; I say that the body shall return into this line with the same velocity, as it had at first.

24. Let AB be prolonged beyond B at pleasure, suppose to G; and from G let GH be drawn, which if produced should always continue equidistant from BF, or, according to the more usual phrase, let GH be drawn parallel to BF. Then it appears, from what has been said upon the second law of motion^a, that in the time, wherein the body would have moved from B to G, had it not received a new impulse in B, by the means of that impulse it will have acquired a velocity, which will carry it from B to H. After the same manner, if CI be

^a Ch. 2. §. 22

taken equal to BH , and IK be drawn equidistant from or parallel to CF ; the body will have moved from C to K with the velocity, which it has in the line CD , in the same time, as it would have employed in moving from C to I with the velocity, it had in the line BC . Therefore since CI and BH are equal, the body will move through CK in the same time, as it would have taken up in moving from B to G with the original velocity, wherewith it moved through the line AB . Again, DL being taken equal to CK and LM drawn parallel to DF ; for the same reason as before the body will move through DM with the velocity, which it has in the line DE , in the same time, as it would employ in moving through BG with its original velocity. In the last place, if EN be taken equal to DM , and NO be drawn parallel to EF ; likewise if AP be taken equal to EO , and PQ be drawn parallel to AF : then the body with the velocity, wherewith it returns into the line AB , will pass through AQ in the same time, as it would have employed in passing through BG with its original velocity. Now as all this follows directly from what has above been delivered, concerning the effect of oblique impulses impressed upon bodies in motion; so we must here observe farther, that it can be proved by geometry, that AQ will always be equal to BG . The proof of this I am obliged, from the nature of my present design, to omit; but this geometrical proposition being granted, it follows, that the body has returned into the line AB with the velocity, which it had, when it first moved in that line; for the velocity, with which it returns into the line AB , will carry it over the line AQ in the same time, as would

have been taken up in its passing over an equal line BG with the original velocity.

25. THUS we have found, how a body may be carried round the figure ABCDE by the action of certain impulses upon it, which should all be pointed toward one center. And we likewise see, that when the body is brought back again to the point, whence it first set out, if it there meet with an impulse sufficient to turn it again into the line, wherein it moved at first, its original velocity will be ~~again~~ restored; and by the repetition of the same impulses, the body will be carried again in the same round. Therefore if these impulses, which act on the body at the points B, C, D, E, and A, continue always the same, the body will make round this figure innumerable revolutions.

26. THE proof, which we have here made use of, holds the same in any number of straight lines, whereof the figure ABD should be composed; and therefore by the method of reasoning referred to above we are to conclude, that what has here been said upon this rectilinear figure, will remain true, if this figure were changed into one of a continued curvature, and ~~instead of~~ distinct impulses acting by intervals at the angles of this figure, we had a continual centripetal force. We have therefore shewn, that a body may be carried round in any curve figure ABC (fig. 82.) which shall every where be concave towards any one point as D, by the continual action

of a centripetal power directed to that point, and when it is returned to the point, from whence it set out, it shall recover again the velocity, with which it departed from that point. It is not indeed always necessary, that it should return again into its first course; for the curve line may have some such figure as the line ABCDBE in fig. 83. In this curve line, if the body set out from B in the direction BF, and moved through the line BCD, but it returned to B; here the body would not enter again into the line BCD, because the two parts BD and BC of the curve line make an angle at the point B: so that the centripetal power, which at the point B could turn the body from the line BF into the curve, will not be able to turn the body into the line BC from the direction, in which it returns to the point B; a forceable impulse must be given the body in the point B to produce that effect.

27. If at the point B, whence the body sets out the curve line return into it self (as in fig. 82;) then the body, upon its arrival again at B, may return into its former course, and thus make an endless circuit about the center of the centripetal power.

28. WHAT has here been said, I hope, will in some measure enable my readers to form a just idea of the nature of these centripetal motions.

29. I HAVE not attempted to shew, how to find particularly, what kind of centripetal force is necessary to carry a body in any curve line proposed. This is to be deduced from the degree

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reasoning. However I shall speak a little to the first proposition, which Sir ISAAC NEWTON lays down for this purpose. By this proposition, when a body is found moving in a curve line, it may be known, whether the body be kept in its course by a power always pointed toward the same center; and if it be so, where that center is placed. The proposition is this: that if a line be drawn from some fixed point to the body, and remaining by one extremum united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its course, be always pointed to this fixed point as a center, this line will move over equal spaces in equal portions of time. Suppose a body were moving through the curve line ABCD (in fig. 84.) and passed over the arches AB, BC, CD in equal portions of time; then if a point, as E, can be found, whence the line EA being drawn to the body in A, and accompanying the body in its motion, it shall make the spaces EAB, EBC, and ECD equal, over which it passes, while the body describes the arches AB, BC, and CD: and if this hold the same in all other arches, both great and small, of the curve line ABCD, that these spaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. THE principle, upon which Sir ISAAC NEWTON has demonstrated this, requires but small skill in geometry to comprehend. I shall therefore take the liberty to close the pre-

sent chapter with an explication of it; because such an example will give the clearest notion of our author's method of applying mathematical reasoning to these philosophical subjects.

31. HE reasons thus. Suppose a body set out from the point A (in fig. 85.) to move in the straight line AB; and after it had moved for some time in that line, it were to receive an impulse directed to some point as C. Let it receive that impulse at D; and thereby be turned into the line DE; and let the body after this impulse take the same length of time in passing from D to E, as it employed in the passing from A to D. Then the straight lines CA, CD, and CE being drawn, Sir ISAAC NEWTON proves, that the and triangular spaces CAD and CDE are equal. This he does in the following manner.

32. LET EF be drawn parallel to CD. Then, from what has been said upon the second law of motion^a, it is evident, that since the body was moving in the line AB, when it received the impulse in the direction DC; it will have moved after that impulse through the line DE in the same time, as it would have taken up in moving through DF, provided it had received no disturbance in D. But the time of the body's moving from D to E is supposed to be equal to the time of its moving through AD; therefore the time, which the body would have employed in moving through DF, had it not been disturbed in D, is equal to the time, wherein it moved through AD: consequently DF is equal in length to AD; for if the

^a Ch. 1. sect. 21, 22.

body had gone on to move through the line AB without interruption, it would have moved through all parts thereof with the same velocity, and have passed over equal parts of that line in equal portions of time. Now CF being drawn, since AD and DF are equal, the triangular space CDF is equal to the triangular space CAD . Farther, the line EF being parallel to CD , it is proved by EUCLID, that the triangle CED is equal to the triangle CDF : therefore the triangle CED is equal to the triangle CAD .

33. AFTER the same manner, if the body receive at E another impulse directed toward the point C , and be turned by that impulse into the line EG ; if it move afterwards from E to G in the same space of time, as was taken up by its motion from D to E , or from A to D ; then CG being drawn, the triangle CEG is equal to CDE . A third impulse at G directed as the two former to C , whereby the body shall be turned into the line GH , will have also the like effect with the rest. If the body move over GH in the same time, as it took up in moving over EG , the triangle CGH will be equal to the triangle CEG . Lastly, if the body at H be turned by a fresh impulse directed toward C into the line HI , and at I by another impulse directed also to C be turned into the line IK ; and if the body move over each of the lines HI , and IK in the same time, as it employed in moving over each of the preceding lines AD , DE , EG , and GH : then each of the triangles CHI , and CIK will be equal to each of the preceding. Like-

wife as the time, in which the body moves over ADE, is equal to the time of its moving over EGH, and to the time of its moving over HIK; the space CADE will be equal to the space CEGH, and to the space CHIK. In the same manner as the time, in which the body moved over ADEG is equal to the time of its moving over GHIK, so the space CADEG will be equal to the space CGHIK.

34. FROM this principle Sir ISAAC NEWTON demonstrates the proposition mentioned above, by that method of arguing introduced by him into geometry, whereof we have before taken notice^a, by making according to the principles of that method a transition from this incurvated figure composed of straight lines, to a figure of continued curvature; and by shewing, that since equal spaces are described in equal times in this present figure composed of straight lines, the same relation between the spaces described and the times of their description will also have place in a figure of one continued curvature. He also deduces from this proposition the reverse of it; and proves, that whenever equal spaces are continually described; the body is acted upon by a centripetal force directed to the center, at which the spaces terminate.

^a § 12.

CHAP. IV.

Of the RESISTANCE of FLUIDS.

BEFORE the cause can be discovered, which keeps the planets in motion, it is necessary first to know, whether the space, wherein they move, is empty and void, or filled with any quantity of matter. It has been a prevailing opinion, that all space contains in it matter of some kind or other; so that where no sensible matter is found, there was yet a subtle fluid substance by which the space was filled up; even so as to make an absolute plenitude. In order to examine this opinion, Sir ISAAC NEWTON has largely considered the effects of fluids upon bodies moving in them.

2. THESE effects he has reduced under these three heads. In the first place he shews how to determine in what manner the resistance, which bodies suffer, when moving in a fluid, gradually increases in proportion to the space, they describe in any fluid; to the velocity, with which they describe it; and to the time they have been in motion. Under the second head he considers what degree of resistance different bodies moving in the same fluid undergo, according to the different proportion between the density of the fluid and the density of the body. The densities of bodies, whether fluid or solid, are measured by the quantity of matter, which is comprehended under the same magnitude; that body being the

the most dense or compact, which under the same bulk contains the greatest quantity of solid matter, or which weighs most, the weight of every body being observed above to be proportional to the quantity of matter in it ^a. Thus water is more dense than cork or wood, iron more dense than water, and gold than iron. The third particular Sir IS. NEWTON considers concerning the resistance of fluids is the influence, which the diversity of ~~figure~~ in the resisted body has upon its resistance.

3. FOR the more perfect illustration of the first of these heads, he distinctly shews the relation between all the particulars specified upon three different suppositions. The first is, that the same body be resisted more or less in the simple proportion to its velocity; so that if its velocity be doubled, its resistance shall become threefold. ^a The second is of the resistance increasing in the duplicate proportion of the velocity; so that, if the velocity of a body be doubled, its resistance shall be rendered four times; and if the velocity be trebled, nine times as great as at first. But what is to be understood by duplicate proportion has been already explained ^b. The third supposition is, that the resistance increases partly in the single proportion of the velocity, ~~and partly in~~ the duplicate proportion thereof.

4. IN all these suppositions, bodies are considered under two respects, either as moving, and opposing themselves

^a Ch. 1. § 24.

^b Ch. 2. sect. 17.

against the fluid by that power alone, which is essential to them, of resisting to the change of their state from rest to motion, or from motion to rest, which we have above called their power of inactivity; or else, as descending or ascending, and so having the power of gravity combined with that other power. Thus our author has shewn in all those three suppositions, in what manner bodies are resisted in an uniform fluid, when they move with the aforesaid progressive motion^a; and what the resistance is, when they ascend or descend perpendicularly^b. And if a body ascend or descend obliquely, and the resistance be singly proportional to the velocity, it is shewn how the body is resisted in a fluid of an uniform density, and what line it will describe^c; which is determined by the measurement of the hyperbola, and appears to be no other than that line, first considered in particular by Dr. BARROW^d, which is now commonly known by the name of the logarithmical curve. In the supposition that the resistance increases in the duplicate proportion of the velocity, our author has not given us the line which would be described in an uniform fluid; but has instead thereof discussed a problem, which is in some sort the reverse; to find the density of the fluid at all altitudes, by which any given curve line may be described; which problem is so treated by him, as to be applicable to any kind of resistance whatever^e. But here not unmindful of practice, he shews that a body in a fluid of uniform density, like the

^a Newt. Princ. Lib. II. prop. 2; 5, 6, 7; 11, 12.
^b ~~Princ. Lib. II. prop. 8, 9; 13, 14.~~
^c Prop. 4.

^d Praelect. Geometr. pag. 123.
^e Newton. Princ. Lib. II. prop. 10.

air, will describe a line, which approaches towards an hyperbola; that is, its motion will be nearer to that curve line than to the parabola. And consequent upon this remark, he shews how to determine this hyperbola by experiment, and briefly resolves the chief of those problems relating to projectiles, which are in use in the art of gunnery, in this curve^a; as TORRICELLI and others have done in the parabola^b. whose inventions have been explained at large above^c.

5. OUR author has also handled distinctly that particular sort of motion, which is described by pendulums^d; and has likewise considered some few cases of bodies moving in resisting fluids round a center, to which they are impelled by a centripetal force, in order to give an idea of those kinds of motions^e.

6. THE treating of the resistance of pendulums has given him an opportunity of inserting into another part of his work some speculations upon the motions of them without resistance, which have a very peculiar elegance; where in he treats of them as moved by a gravitation acting in the law, which he shews to belong to the earth below its surface^f; performing in this kind of gravitation, where the force is proportional to the distance from the center, all that HUYGENS had before done in the common supposition of its being uniform, and acting in parallel lines^g.

^a Newton. Princ. Lib. II. prop. 10. in schol.

^b Torricelli de motu gravium.

^c Ch. 2. § 85, &c.

^d Newt. Princ. Lib. II. sect. 6.

^e Lib. II. sect. 4.

^f See B. II. Ch. 6. § 7. of this treatise.

^g Lib. I. sect. 10.

7. HUYGENS at the end of his treatise of the cause of gravity^a informs us, that he likewise had carried his speculations on the first of these suppositions, of the resistance in fluids being proportional to the velocity of the body, as far as our author. But finding by experiment that the second was more conformable to nature, he afterwards made some progress in that, till he was stopt; by not being able to execute to his wish what related to the perpendicular descent of bodies; not observing that the measurement of the curve line, he made use of to explain it by, depended on the hyperbola. Which oversight may well be pardoned in that great man, considering that our author had not been pleased at that time to communicate to the publick his admirable discourse of the QUADRATURE OF MEASUREMENT OF CURVE LINES, with which he has since obliged the world: for without the use of that treatise, it is I think no injury even to our author's unparalleled abilities to believe, it would not have been easy for himself to have succeeded so happily in this and many other parts of his writings.

8. WHAT HUYGENS found by experiment, that bodies were in reality resisted in the duplicate proportion of their velocity, agrees with the reasoning of our author^b, who distinguishes the resistance, which fluids give to bodies by the tenacity of their parts, and the friction between them and the body, from that, which arises from the power of inactivity, with which the constituent particles of fluids are endued like all

^a De la Pésanteur, pag. 169, and the following. | ^b Newton. Princ. L. II. prop. 4. schol.

other portions of matter, by which power the particles of fluids like other bodies make resistance against being put into motion.

9. THE resistance, which arises from the friction of the body against the parts of the fluid, must be very inconsiderable; and the resistance, which follows from the tenacity of the parts of fluids, is not usually very great, and does not depend much upon the velocity of the body in the fluid; for as the parts of the fluid adhere together with a certain degree of force, the resistance, which the body receives from thence, cannot much depend upon the velocity, with which the body moves; but like the power of gravity, its effect must be proportional to the time of its acting. This the reader may find farther explained by Sir ISAAC NEWTON himself in the postscript to a discourse published by me in THE PHILOSOPHICAL TRANSACTIONS, N^o 371. The principal resistance, which most fluids give to bodies, arises from the power of inactivity in the parts of the fluids, and this depends upon the velocity, with which the body moves, on a double account. In the first place, the quantity of the fluid moved out of place by the moving body in any determinate space of time is proportional to the velocity, wherewith the body moves; and in the next place, the velocity with which each particle of the fluid is moved, will also be proportional to the velocity of the body: therefore since the resistance, which any body makes against being put into motion, is proportional both to the quantity of matter moved and the velocity it is moved with; the resistance, which a fluid gives on this account, will be doubly increased with the increase of the velocity in the moving body; that

that is, the resistance will be in a two-fold or duplicate proportion of the velocity, wherewith the body moves through the fluid.

10. FARTHER it is most manifest, that this latter kind of resistance increasing with the increase of velocity, even in a greater degree than the velocity it self increases, the swifter the body moves, the less proportion the other species of resistance will bear to this: nay that this part of the resistance may be so much augmented by a due increase of velocity, till the former resistances shall bear a less proportion to this, than any that might be assigned. And indeed experience shews, that no other resistance, than what arises from the power of inactivity in the parts of the fluid, is of moment, when the body moves with any considerable swiftness.

11. THERE is besides these yet another species of resistance, found only in such fluids, as, like our air, are elastic. Elasticity belongs to no fluid known to us beside the air. By this property any quantity of air may be contracted into a less space by a forcible pressure, and as soon as the compressing power is removed, it will spring out again to its former dimensions. The air we breath is held to its present density by the weight of the air above us. And as this incumbent weight, by the motion of the winds, or other causes, is frequently varied (which appears by the barometer;) so when this weight is greatest, we breath a more dense air than at other times. To what degree the air would expand it self by its spring, if all pressure were removed, is not known,

known, nor yet into how narrow a compass it is capable of being compressed. Mr. BOYLE found it by experiment capable both of expansion and compression to such a degree, that he could cause a quantity of air to expand it self over a space some hundred thousand times greater, than the space to which he could confine the same quantity ^a. But I shall treat more fully of this spring in the air hereafter ^b. I am now only to consider what resistance to the motion of bodies arises from it.

12. BUT before our author shews in what manner this cause of resistance operates, he proposes a method, by which fluids may be rendered elastic, demonstrating that if their particles be provided with a power of repelling each other, which shall exert it self with degrees of strength reciprocally proportional to the distances between the centers of the particles; that then such fluids will observe the same rule in being compressed, as our air does, which is this, that the space, into which it yields upon compression, is reciprocally proportional to the compressing weight ^c. The term reciprocally proportional has been explained above ^d. And if the centrifugal force of the particles acted by other laws, such fluids would yield in a different manner to compression ^e.

13. WHETHER the particles of the air be endued with such a power, by which they can act upon each other out of contact, our author does not determine; but leaves that

^a See his Tract on the admirable rarification of the air.

^b Book II. Ch. 6.

^c Princ. philos. Lib. II. prop. 23.

^d Book I. Ch. 2. § 30.

^e Princ. philos. Lib. II. prop. 23. in schol.

to future examination, and to be discussed by philosophers. Only he takes occasion from hence to consider the resistance in elastic fluids, under this notion; making remarks, as he passes along, upon the differences, which will arise, if their elasticity be derived from any other fountain ^a. And this, I think, must be confessed to be done by him with great judgment; for this is far the most reasonable account, which has been given of this surprizing power; as must without doubt be freely acknowledged by any one, who in the least considers the insufficiency of all the other conjectures, which have been framed; and also how little reason there is to deny to bodies other powers, by which they may act upon each other at a distance, as well as that of gravity; which we shall hereafter shew to be a property universally belonging to all the bodies of the universe, and to all their parts ^b. Nay we actually find in the loadstone a very apparent repelling, as well as an attractive power. But of this more in the conclusion of this discourse.

14. By these steps our author leads the way to explain the resistance, which the air and such like fluids will give to bodies by their elasticity; which resistance he explains thus. If the elastic power of the fluid were to be varied so, as to be always in the duplicate proportion of the velocity of the resisted body, it is shewn that then the resistance, derived from the elasticity, would increase in the duplicate proportion of the velocity; in so much that the

^a Princ. philos. Lib. II. prop. 33. coroll. | ^b Lib. II. Ch. 5.

whole resistance would be in that proportion, excepting only that small part, which arises from the friction between the body and the parts of the fluid. From whence it follows, that because the elastic power of the same fluid does in truth continue the same, if the velocity of the moving body be diminished, the resistance from the elasticity, and therefore the whole resistance, will decrease in a less proportion, than the duplicate of the velocity; and if the velocity be increased, the resistance from the elasticity will increase in a less proportion, than the duplicate of the velocity, that is in a less proportion, than the resistance made by the power of inactivity of the parts of the fluid. And from this foundation is raised the proof of a property of this resistance, given by the elasticity in common with the others from the tenacity and friction of the parts of the fluid; that the velocity may be increased, till this resistance from the fluid's elasticity shall bear no considerable proportion to that, which is produced by the power of inactivity thereof. From whence our author draws this conclusion; that the resistance of a body, which moves very swiftly in an elastic fluid, is near the same, as if the fluid were not elastic; provided the elasticity arises from the centrifugal power of the parts of the medium, as before explained, especially if the velocity be so great, that this centrifugal power shall want time to exert it self^b. But it is to be observed, that in the proof of all this our author proceeds upon the supposition of this centrifugal power in the parts of the fluid; but if the elasticity be caused by the expansion of the parts in the

^a See Prop. 33. coroll. 2.

^b Ibid. coroll. 3.

manner of wool compressed, and such like bodies, by which the parts of the fluid will be in some measure entangled together, and their motion be obstructed, the fluid will be in a manner tenacious, and give a resistance upon that account over and above what depends upon its elasticity only^a; and the resistance derived from that cause is to be judged of in the manner before set down.

15. It is now time to pass to the second part of this theory; which is to assign the measure of resistance, according to the proportion between the density of the body and the density of the fluid. What is here to be understood by the word density has been explained above^b. For this purpose as our author before considered two distinct cases of bodies moving in mediums; one when they opposed themselves to the fluid by their power of inactivity only, and another when by ascending or descending their weight was combined with that other power: so likewise, the fluids themselves are to be regarded under a double capacity; either as having their parts at rest, and disposed freely without restraint, or as being compressed together by their own weight, or any other cause.

16. In the first case, if the parts of the fluid be wholly disengaged from one another, so that each particle is at liberty to move all ways without any impediment, it is shewn, that if a globe move in such a fluid, and the globe and par-

^a Vid. *ibid.* coroll. 6.

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^b In § 2.

ticles of the fluid are endued with perfect elasticity; so that as the globe impinges upon the particles of it, they shall bound off and separate themselves from the globe, with the same velocity, with which the globe strikes upon them; then the resistance, which the globe moving with any known velocity suffers, is to be thus determined. From the velocity of the globe, the time, wherein it would move over two third parts of its own diameter with that velocity, will be known. And such proportion as the density of the fluid bears to the density of the globe, the same the resistance given to the globe will bear to the force, which acting, like the power of gravity, on the globe without intermission during the space of time now mentioned, would generate in the globe the same degree of motion, as that wherewith it moves in the fluid^a. But if neither the globe nor the particles of the fluid be elastic, so that the particles, when the globe strikes against them, do not rebound from it, then the resistance will be but half so much^b. Again, if the particles of the fluid and the globe are imperfectly elastic, so that the particles will spring from the globe with part only of that velocity wherewith the globe impinges upon them; then the resistance will be a mean between the two preceding cases, approaching nearer to the first or second, according as the elasticity is more or less^c.

17. THE elasticity, which is here ascribed to the particles of the fluid, is not that power of repelling one another,

^a Princ. philos. Lib. II. Prop. 35. |
^b Ibid.

^c Id.

when out of contact; by which, as has before been mentioned, the whole fluid may be rendred elastic; but such an elasticity only, as many solid bodies have of recovering their figure, whenever any forcible change is made in it, by the impulse of another body or otherwise. Which elasticity has been explained above at large ^a.

18. THIS is the case of discontinued fluids, where the body, by pressing against their particles, drives them before itself, while the space behind the body is left empty. But in fluids which are compressed, so that the parts of them removed out of place by the body resisted immediately retire behind the body, and fill that space, which in the other case is left vacant, the resistance is still less; for a globe in such a fluid which shall be free from all elasticity, will be resisted but half as much as the least resistance in the former case ^b. But by elasticity I now mean that power, which renders the whole fluid so; of which if the compressed fluid be possessed, in the manner of the air, then the resistance will be greater than by the foregoing rule; for the fluid being capable in some degree of condensation, it will resemble so far the case of un-compressed fluids ^c. But, as has been before related, this difference is most considerable in flow motions.

19. IN the next place our author is particular in determining the degrees of resistance accompanying bodies of different figures; which is the last of the three heads, we

^a h. 1. § 29.

^b Princ. philos. Lib. II. Prop. 38, compared with

coroll. 1. of prop. 35.

^c L. II. Lem. 7. schol. pag. 341.

divided the whole discourse of resistance into. And in this disquisition he finds a very surprizing and unthought of difference, between free and compressed fluids. He proves, that in the former kind, a globe suffers but half the resistance, which the cylinder, that circumscribes the globe, will do, if it move in the direction of its axis ^a. But in the latter he proves, that the globe and cylinder are resisted alike ^b. And in general, that let the shape of bodies be ever so different, yet if the greatest sections of the bodies perpendicular to the axis of their motion be equal, the bodies will be resisted equally ^c.

20. PURSUANT to the difference found between the resistance of the globe and cylinder in rare and uncompressed fluids, our author gives us the result of some other inquiries of the same nature. Thus of all the frustums of a cone, that can be described upon the same base and with the same altitude, he shews how to find that, which of all others will be the least resisted, when moving in the direction of its axis ^d. And from hence he draws an easy method of altering the figure of any spheroidical solid, so that its capacity may be enlarged, and yet the resistance of it diminished ^e: a note which he thinks may not be useless to shipwrights. He concludes with determining the solid, which will be resisted the least that is possible, in these discontinued fluids ^f.

^a Lib. II. Prop. 34.

^b Lib. II. Lem. 7. p. 341.

^c Schol. to Lem. 7.

^d Prop. 34. schol.

^e Ibid.

^f Ibid.

21. ~~THAT~~ I may here be understood by readers unacquainted with mathematical terms, I shall explain what I mean by a frustum of a cone, and a spheroidal solid. A cone has been defined above. A frustum is what remains, when part of the cone next the vertex is cut away by a section parallel to the base of the cone, as in fig. 86. A spheroid is produced from an ellipsis, as a sphere or globe is made from a circle. If a circle turn round on its diameter, it describes by its motion a sphere; so if an ellipsis (which figure has been defined above, and will be more fully explained hereafter^a) be turned round either upon the longest or shortest line, that can be drawn through the middle of it, there will be described a kind of oblong or flat sphere; as in fig. 87. Both these figures are called spheroids, and any solid resembling these I here call spheroidal.

22. If it should be asked, how the method of altering spheroidal bodies, here mentioned, can contribute to the facilitating a ship's motion, when I just above affirmed, that the figure of bodies, which move in a compressed fluid not elastic, has no relation to the augmentation or diminution of the resistance; the reply is, that what was there spoken relates to bodies deep immersed into such fluids, but not of those, which swim upon the surface of them; for in this latter case the fluid, by the appulse of the anterior parts of the body, is raised above the level of the surface, and behind the body is sunk somewhat below; so

^a Book II, Ch. 1. § 6,

that by this inequality in the superficies of the fluid, that part of it, which at the head of the body is higher than the fluid behind, will resist in some measure after the manner of discontinued fluids^a, analogous to what was before observed to happen in the air through its elasticity, though the body be furrounded on every side by it^b. And as far as the power of these causes extends, the figure of the moving body affects its resistance; for it is evident, that the figure, which presses least directly against the parts of the fluid, and so raises least the surface of a fluid not elastic, and least compresses one that is elastic, will be least resisted.

23. THE way of collecting the difference of the resistance in rare fluids, which arises from the diversity of figure, is by considering the different effect of the particles of the fluid upon the body moving against them, according to the different obliquity of the several parts of the body upon which they respectively strike; as it is known, that any body impinging against a plane obliquely, strikes with a less force, than if it fell upon it perpendicularly; and the greater the obliquity is, the weaker is the force. And it is the same thing, if the body be at rest, and the plane move against it^c.

24. THAT there is no connexion between the figure of a body and its resistance in compressed fluids, is proved thus. Suppose ABCD (in fig. 88.) to be a canal, having such a fluid, water for instance, running through it with an equable

^a Vid. Newt. princ. in schol. to Lem. 7, of Lib. II. pag. 341. | ^b Sect. 17. of this chapter. | ^c See Princ. philos. Lib. II. prop. 34.

velocity;

velocity; and let any body E, by being placed in the axis of the canal, hinder the passage of the water. It is evident, that the figure of the fore part of this body will have little influence in obstructing the water's motion, but the whole impediment will arise from the space taken up by the body, by which it diminishes the bore of the canal, and straightens the passage of the water^a. But proportional to the obstruction of the water's motion, will be the force of the water upon the body E^b. Now suppose both orifices of the canal to be closed, and the water in it to remain at rest; the body E to move, so that the parts of the water may pass by it with the same degree of velocity, as they did before; it is beyond contradiction, that the pressure of the water upon the body, that is, the resistance it gives to its motion, will remain the same; and therefore will have little connexion with the figure of the body^c.

25. By a method of reasoning drawn from the same fountain is determined the measure of resistance these compressed fluids give to bodies, in reference to the proportion between the density of the body and that of the fluid. This shall be explained particularly in my comment on Sir I S. NEWTON'S mathematical principles of natural philosophy; but is not a proper subject to be insisted on farther in this place.

26. WE have now gone through all the parts of this theory. There remains nothing more, but in few words to mention the experiments, which our author has made, both

^a Vid. Princ. philos. Lib. II. Lem. 5. p. 314. [^b *Idem* 6. [^c *Ibid*. 7.

with bodies falling perpendicularly through water, and the air^a, and with pendulums^b: all which agree with the theory. In the case of falling bodies, the times of their fall determined by the theory come out the same, as by observation, to a surprizing exactness; in the pendulums, the rod, by which the ball of the pendulum hangs, suffers resistance as well as the ball, and the motion of the ball being reciprocal, it communicates such a motion to the fluid, as increases the resistance; but the deviation from the theory is no more, than what may reasonably follow from these causes.

27. By this theory of the resistance of fluids, and these experiments, our author decides the question so long agitated among natural philosophers, whether all space is absolutely full of matter. The Aristotelians and Cartesians both assert this plenitude; the Atomists have maintained the contrary. Our author has chose to determine this question by his theory of resistance, as shall be explained in the following chapter.

^a Newt. Princ. Lib. II. prop. 40. in schol.

|

^b Lib. II. in schol. post prop. 31.



BOOK

